Name: ______________________________

Math 3310
Exam 2
Spring 2018

Department of Math and Stat
Texas Tech University

Part I

Solve problems 1–5 below; they count for 8% each. Your answers must be written on this sheet of paper. No aids are allowed on this part of the test. When you have turned in this part, you may use books and notes to solve the problems on Part II.

1. Mark with a ✓ each statement that is proved by the following argument.

   Set $n = 2k$. One then has
   $$n(n+1) = 2k(2k+1) = 2(2k^2+k)\ ,$$
   and as $(2k^2+k)$ is an integer, $n(n+1)$ is even.

   (a) For every integer n, the number $n(n+1)$ is even.

   (b) If $n(n+1)$ is odd, then so is n.

   (c) If n is odd, then $n(n+1)$ is even.

   (d) If n is even, then $n(n+1)$ is even.

2. Pair up each statement on the left with its contrapositive statement on the right.

Let $m \in \mathbb{Z}$. If m is odd, then m^2 is odd.	Let $m \in \mathbb{Z}$. If m^2 is odd, then m is even.
Let $m \in \mathbb{Z}$. If m is even, then m^2 is even.	Let $m \in \mathbb{Z}$. If m^2 is odd, then m is odd.
Let $m \in \mathbb{Z}$. If m is even, then m^2 is odd.	Let $m \in \mathbb{Z}$. If m^2 is even, then m is even.
Let $m \in \mathbb{Z}$. If m is odd, then m^2 is even.	Let $m \in \mathbb{Z}$. If m^2 is even, then m is odd.

3. Mark with a ✓ each statement that is proved by the following argument.

   Let x and y be positive real numbers. Assume that $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$ holds, one then has $x + y = (\sqrt{x} + \sqrt{y})^2 = x + y + 2\sqrt{xy}$, which implies $0 = \sqrt{xy}$ in contradiction of the assumption.

   (a) For positive real numbers x and y, one cannot take the square root of $x + y$.

   (b) There are no positive real numbers x and y with $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$.

   (c) There are no real numbers x and y with $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$.

   (d) For real numbers x and y one has $\sqrt{x+y} \leq \sqrt{x} + \sqrt{y}$.

4. Formulate a statement that is proved by the following argument.

> Let $p_1, p_2, \ldots, p_n$ be primes and set $P = p_1 p_2 \cdots p_n + 1$. Let $i \in \{1, \ldots, n\}$, if p_i divides P, then p_i divides $1 = P - p_1 p_2 \cdots p_n$, which is absurd. Thus, P is not divisible by any of the primes $p_1, p_2, \ldots, p_n$. As P is divisible by some prime, it follows that there exists a prime p different from the primes $p_1, p_2, \ldots, p_n$.

5. Each of the next two statements can be proved by contraposition. Write the beginning of such an argument, not the whole argument, just the opening sentence with the assumption you make.

   (a) Let m be an integer. If $3m - 8$ is even, then m is even.

   (b) Let x be a real number. If every odd power of x is negative, then x is negative.

Part II

Solve five (5) of the problems 6–11; they count for 12% each. Your solutions to these problems must be written on blank pages or in a blue book. Turn in your solutions to five problems only!

Books, notes, and old homework are allowed aids on this part of the test. Calculators are also allowed, but phones, PDAs, Apple watches etc. are not. For full credit, you must show complete, correct, legible work. Read carefully before you start working.

6. Prove the following result.

   Let m be an integer. If m is odd, then $m(m-2)$ is odd.

7. Prove the following result.

   Let m be an integer. If 3 does not divide m, then 3 does not divide $m^3 + m$.

8. Prove the following result.

   For all real numbers x and y one has $|xy| = |x||y|$.

9. Prove the following result.

   Let m and n be natural numbers. If $mn > 100$, then one has $m > 10$ or $n > 10$.

10. For each of the following statements, prove whether it is a contradiction, a tautology, or neither.

    (a) Let m be an integer, m is odd or m is even.

    (b) Let m be an integer. If m is odd, then $m+1$ is divisible by 4.

    (c) Let m be an integer, m is odd and $-m$ is even.

11. Show that the equation $x^2 + 5x + 5 = 0$ has a unique solution in the interval $(-4, -3)$.