

Part I

Solve problems 1–5 below; they count for 8% each. Your answers must be written on this sheet of paper. No aids are allowed on this part of the test. When you have turned in this part, you may use books and notes to solve the problems on Part II.

1. Mark with a \checkmark the collections of subsets that are partitions of the interval $[0, \infty)$.
 - (a) $\{[n - 1, n) \mid n \in \mathbb{N}\}$
 - (b) $\{[n - \frac{1}{2}, n + \frac{1}{2}) \mid n \in \mathbb{N}\}$
 - (c) $\{\{x\} \mid x \in [0, \infty)\}$
 - (d) $\{[0, q) \mid q \in \mathbb{Q}\}$

2. Consider the sets $A = \{1, 3, 5\}$, $B = \{5, 3\}$, and $C = \{1, 2, 4\}$. For each of the following statements, decide if it is true (T) or false (F).
 - (a) $1 \in A$
 - (b) $\{3\} \subseteq A$
 - (c) $\{5\} \in A$
 - (d) $A \subseteq B$
 - (e) $\emptyset \in C$
 - (f) $A \cap C \subseteq B$
 - (g) $A \cap C \subseteq A - B$
 - (h) $B \times C \subseteq A \times C$

3. Consider the intervals given by $I = [-15, 5]$, $J = (9, 27)$, and $K = (15, 25)$. Mark with a \checkmark the sets below that contain the number 20 as an element.
 - (a) $I \cap J$
 - (b) \bar{I}
 - (c) $(I \cup J) \cap K$
 - (d) $\overline{I \cap (J \cup K)}$

4. Mark with a \checkmark each of the following sentences that is a statement.

- (a) Every real number is rational.
- (b) Assume that x is a rational number.
- (c) Is p is a prime number, then p is odd.
- (d) For some real number x , the square root of x is irrational.

5. Complete the truth table

| P | Q | $\sim P$ | $\sim Q$ | $P \Rightarrow Q$ | $(P \Rightarrow Q) \wedge \sim Q$ |
|-----|-----|----------|----------|-------------------|-----------------------------------|
| T | T | | | | |
| T | F | | | | |
| F | T | | | | |
| F | F | | | | |

and use it to decide if the statement

$$(P \Rightarrow Q) \wedge \sim Q \implies \sim P$$

is a tautology.

Part II

Solve five (5) of the problems 6–11; they count for 12% each. Your solutions to these problems must be written on blank pages or in a blue book. Turn in your solutions to five problems only!

Books, notes, and old homework are allowed aids on this part of the test. Calculators are also allowed, but phones, PDAs, Apple watches etc. are not. For full credit, you must show complete, correct, legible work. Read carefully before you start working.

6. Describe the following sets by listing their elements.

- (a) $\emptyset \cap \{\emptyset\}$
- (b) $\emptyset \cup \{\emptyset\}$
- (c) $\{\emptyset\} \times \{\emptyset\}$
- (d) $\mathcal{P}(\{1, 2, 3\}) - \mathcal{P}(\{3, 4\})$

7. Find the (unique) partition \mathcal{S} of \mathbb{Z} that satisfies the following requirements:

- (a) $|\mathcal{S}| = 2$
- (b) $\exists A \in \mathcal{S}, |A| = 1$
- (c) $\forall A \in \mathcal{S}, \forall a \in A, -a \in A$
- (d) $\forall A \in \mathcal{S}, \forall a \in A, a^2 \in A$

8. Write the next statements in words.

- (a) $\exists x \in \mathbb{R}, x^2 \in \mathbb{N}$.
- (b) $\forall n \in \mathbb{Z}, \cos(n\pi) \in \mathbb{Z}$.
- (c) $\exists x \in \mathbb{R} - \mathbb{Q}, \exists y \in \mathbb{R} - \mathbb{Q}, x^y \in \mathbb{Q}$.

9. Negate each of the next statements.

- (a) Every ellipse is a circle.
- (b) 23 and 53 are prime numbers.
- (c) There is a rational number q such that the number \sqrt{q} is not rational.

10. Negate each of the next statements.

- (a) $\forall q \in \mathbb{Q}, \exists n \in \mathbb{N}, nq \in \mathbb{Z}$.
- (b) If 2 is prime, then 2 is odd.
- (c) $\sqrt{2}$ is rational or $\sqrt{4}$ is rational.

11. For statements P , Q , and R show that

$$((P \wedge Q) \Rightarrow R) \iff (P \Rightarrow (Q \Rightarrow R))$$

is a tautology.