Name: ______________________________

Math 3310
Exam 1
Fall 2018

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Part I

Solve problems 1–5 below; they count for 8% each. Your answers must be written on this sheet of paper. No aids are allowed on this part of the test. When you have turned in this part, you may use books and notes to solve the problems on Part II.

1. Given the sets $X = \{a, b\}$ and $Y = \{a, \{a, b\}, c\}$, construct the following sets:

   (a) $X \times Y$.

   (b) $Y - X$.

2. Create a partition $\mathcal{S}$ of the set of integers $\mathbb{Z}$ that satisfies both of the following two conditions:

   - $|\mathcal{S}| = 3$
   - $\forall\ X \in \mathcal{S}, \forall n \in \mathbb{Z}$, if $n \in X$, then $-n \in X$.

3. Mark with a ✓ the collections of subsets that are partitions of the interval $(1, \infty)$.

   (a) $\{(n, n+1] \mid n \in \mathbb{N}\}$

   (b) $\{(|n|, |n|+1] \mid n \in \mathbb{Z}\}$

   (c) $\{(|n|+1, |n|+2] \mid n \in \mathbb{Z}\}$

   (d) $\{(n, n+1) \mid n \in \mathbb{N}\}$

4. Pair up each statement on the left with its contrapositive statement on the right.

Let $m \in \mathbb{Z}$. If m is odd, then m^2 is odd.	Let $m \in \mathbb{Z}$. If m^2 is odd, then m is even.
Let $m \in \mathbb{Z}$. If m is even, then m^2 is even.	Let $m \in \mathbb{Z}$. If m^2 is odd, then m is odd.
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Let $m \in \mathbb{Z}$. If m is odd, then m^2 is even.	Let $m \in \mathbb{Z}$. If m^2 is even, then m is odd.

5. Complete the truth table

P	Q	$P \Rightarrow Q$	$P \wedge Q$	$(P \wedge Q) \Leftrightarrow P$
T	T			
T	F			
F	T			
F	F			

and use it to decide if the statements $P \Rightarrow Q$ and $(P \wedge Q) \iff P$ are logically equivalent.

Part II

Solve five (5) of the problems 6–11; they count for 12% each. Your solutions to these problems must be written on blank pages or in a blue book. Turn in your solutions to five problems only!

Books, notes, and old homework are allowed aids on this part of the test. Calculators are also allowed, but phones, PDAs, Apple watches etc. are not. For full credit, you must show complete, correct, legible work. Read carefully before you start working.

6. Consider the two statements

$$P\colon 5 \text{ is a prime number} \quad \text{and} \quad Q\colon \{2n : n \in \mathbb{Z}\} \cap \{2n+1 : n \in \mathbb{Z}\} \neq \varnothing\,.$$

For each of the statements (a)–(f) below, determine if it is true or false and explain why.

(a) $P \vee Q$
(b) $\sim P \implies \sim Q$
(c) $P \vee (\sim Q)$
(d) $P \wedge Q$
(e) $(\sim P) \wedge Q$
(f) $(\sim P) \vee (\sim Q)$

7. Let $A = \{x, y, z\}$, $B = \{x, y, w, a, b, c\}$, $C = \{a, \{x\}, d, \varnothing\}$, and $D = \{x, y, z, w\}$ be sets. For each of the statements (a)–(f) about these sets, decide if it is true or false and explain why.

(a) $A \subseteq B$
(b) $\varnothing \subseteq B$
(c) $x \in A \cap B \cap C \cap D$
(d) $\{\varnothing\} \notin \mathcal{P}(C)$
(e) $(z, w) \in B \times D$
(f) $(\varnothing, \varnothing) \in B \times C$

8. Write the next statements in words.

(a) $\forall x \in \mathbb{R} - \mathbb{Q},\ \forall y \in \mathbb{Q},\ x + y \notin \mathbb{Q}$.
(b) $\forall n \in \mathbb{Z},\ \exists\, m \in \mathbb{Z},\ m = \sqrt{n^2}$.
(c) $\exists\, z \in \mathbb{R},\ \forall u \in \mathbb{R},\ \forall v \in \mathbb{R},\ zu = zv$.

9. Negate each of the next statements.

   (a) For every real number x one has $x = \sqrt{x^2}$.

   (b) At least one of the numbers 23, 53, or 141 is not prime.

   (c) There is no rational number q such that the number $\sqrt{q}$ is not rational.

10. Prove the following result.

    Let m be an integer. If m is odd, then $m(m-2)$ is odd.

11. Prove the following result.

    Let m be an integer. If $3m-8$ is odd, then m odd.