NAME: Key

MATH 3310
QUIZ 2
FALL 2017

DEPARTMENT OF MATH AND STAT
TEXAS TECH UNIVERSITY

1. Each of the following arguments contains an error. Mark it and use the blank line to explain what the problem is.

   (a) Let x and y be integers. If $x = y$, then one has $xy = y^2$ and hence $x^2 - xy = x^2 - y^2$. This may be rewritten as $x(x-y) = (x+y)(x-y)$, and cancellation of common factors yields $x = x + y$. Thus, for $x = 1 = y$ one gets $1 = 2$.

   Cancellation only valid for $x \neq y$ as div by 0 not def.

   (b) For every $\theta \in [0, 2\pi)$ the Pythagorean Theorem yields $\cos^2\theta + \sin^2\theta = 1$ and hence $\cos\theta = \sqrt{1 - \sin^2\theta}$. Evaluating this expression at $\theta = \pi$ one gets $-1 = \sqrt{1-0} = 1$.

   $\sqrt{\cos^2\theta} = |\cos\theta|$

   (c) If a is even and b is odd, then $a + b - 1$ is divisible by 4. Indeed, one has $a = 2k$ and $b = 2k + 1$, so $a + b - 1 = 2k + (2k + 1) - 1 = 4k$.

   Argument only valid for $b = a+1$

   (d) Let $p_1, p_2, \ldots, p_n$ be primes. Since $P = p_1 p_2 \cdots p_n + 1$ is not divisible by any of the primes $p_1, p_2, \ldots, p_n$ it must itself be prime, and there are thus infinitely many primes.

   Could be divisible by a prime $q \neq p_i$ for $i = 1, \ldots, n$.
   $3 \cdot 5 + 1 = 16$ div. by 2.

2. For each of the following arguments, decide what is being proved.

   (a) Let n be even and write $n = 2k$. One then has
   $$n^2 - n = (2k)^2 - 2k = 2(2k^2 - k),$$
   and as $2k^2 - k$ is an integer, $n^2 - n$ is even.

      i. If $n^2 - n$ is odd, then so is n.
      ii. If n is odd, then so is $n^2 - n$.
      iii. For every integer n, the number $n^2 - n$ is even.
      iv. If n is even, then $n^2 - n$ is even.

   (b) Set $n = 2k + 1$. One then has
   $$n(n+1) = (2k+1)((2k+1)+1) = (2k+1)(2k+2) = 2(2k+1)(k+1),$$
   and as $(2k+1)(k+1)$ is an integer, $n(n+1)$ is even.

      i. For every integer n, the number $n(n+1)$ is even.
      ii. If $n(n+1)$ is odd, then so is n.
      iii. If n is odd, then $n(n+1)$ is even.
      iv. If n is even, then $n(n+1)$ is even.

   (c) The numbers 2 and 11 are prime, but $2(11) - 1 = 22 - 1 = 21 = 3(7)$ is not prime.

      i. For odd primes p and q, the number $pq - 1$ is not a prime.
      ii. Nothing
      iii. It is not true that $pq - 1$ is prime for all primes p and q.
      iv. There exist primes p and q, such that $pq - 1$ is not a prime.

   (d) Let α be the repeated decimal $0.\overline{99}$. One has $10\alpha = 9.\overline{99}$ and, therefore, $9\alpha = 10\alpha - \alpha = 9$. That is, $\alpha = 1$.

      i. Nothing
      ii. The repeated decimal $0.\overline{99}$ is the number 1.
      iii. Not every number has a unique decimal representation.
      iv. One has $1.\overline{00} = 0.\overline{99}$.