CHANGE OF BASIS

Let V and W be vector spaces, and let $T\colon V \longrightarrow W$ be a linear transformation. Given bases $\mathcal{S}$ and $\mathcal{B}$ for V and W, the transformation T is represented by a matrix A; that is

$$A[v]_{\mathcal{S}} = [T(v)]_{\mathcal{B}} \ .$$

For another choice of bases, $\mathcal{S}'$ and $\mathcal{B}'$, the transformation is represented by a matrix A'. If P is the transition matrix from $\mathcal{S}'$ to $\mathcal{S}$ and Q is the transition matrix from $\mathcal{B}'$ to $\mathcal{B}$, i.e.

$$[v]_{\mathcal{S}} = P[v]_{\mathcal{S}'} \qquad \text{and} \qquad [w]_{\mathcal{B}} = Q[w]_{\mathcal{B}'} \ ,$$

then A' is determined by A via the formula

$$A' = Q^{-1}AP \ .$$