

## MATH 2360-D01 WEEK 13

SECTIONS 5.1–5.3; PAGES 225–258

**ABSTRACT.** The dot product of vectors in  $\mathbb{R}^2$  and  $\mathbb{R}^3$  generalizes to  $\mathbb{R}^n$ , so one can speak of length and orthogonality of vectors in  $\mathbb{R}^n$ . The dot product on  $\mathbb{R}^n$  is just one example of an inner product on a vector space. In any vector space with an inner product, one can talk about length and orthogonality, and given a basis for such a space, there is a procedure to produce a basis with pairwise orthogonal vectors of unit length.

### SECTION 5.1

**Reading.** Make sure that you understand the following:

- (1) The well-known dot product on  $\mathbb{R}^2$  and  $\mathbb{R}^3$  has a straightforward generalization to  $\mathbb{R}^n$  for any  $n \geq 1$ .
- (2) All the notions for vectors in  $\mathbb{R}^2$  and  $\mathbb{R}^3$  that are based on the dot product carry over to  $\mathbb{R}^n$ . They include length of vectors, angle between vectors, and distance between vectors.
- (3) The Triangle Inequality and the Pythagorean Theorem also carry over to  $\mathbb{R}^n$ .

**Suggested problems.** To verify that you have understood the material, solve the following problems at the end of the section: 1, 3, 7, 11, 21, 23, and 25.

### SECTION 5.2

**Reading.** Make sure that you understand the following:

- (1) That any inner product on a vector space has the same essential properties as the dot product on  $\mathbb{R}^n$  and gives definitions of length, distance, angle, etc.
- (2) What projection of vectors means in an inner product space.

**Suggested problems.** To verify that you have understood the material, solve the following problems at the end of the section: 3, 5, 11, 19, and 39.

### SECTION 5.3

**Reading.** Make sure that you understand the following:

- (1) What it means for a set of vectors to be orthogonal.
- (2) What it means for a set of vectors to be orthonormal.
- (3) That an orthogonal set of vectors is linearly independent.
- (4) The Gram–Schmidt orthonormalization process.

**Suggested problems.** To verify that you have understood the material, solve the following problems at the end of the section: 3, 7, 11, 15, and 31.