# MATH 2360-D01 WEEK 10

SECTIONS 6.1 AND 6.2; PAGES 291–313

ABSTRACT. A *linear transformation* is a simple, but extremely useful, type of function. It maps from one vector space to another and respects the addition and multiplication by scalars. Every matrix defines a linear transformation.

## SECTION 6.1

**Reading.** Make sure that you understand the following:

(1) The definition of a *linear transformation.*

(2) That an $m \times n$ matrix $A$ defines a linear transformation $T\colon \mathbb{R}^n \to \mathbb{R}^m$ given by $T(\mathbf{x}) = A\mathbf{x}$ for vectors $\mathbf{x} \in \mathbb{R}^n$.

**Suggested problems.** To verify that you have understood the material, solve the following problems at the end of the section: 1–4, 10, 16, 22, and 36.

## SECTION 6.2

**Reading.** Make sure that you understand the following:

(1) Given a linear transformation $T\colon V \to W$, the *kernel* of $T$ is a subspace of $V$ and the *range*—also called the *image*—of $T$ is a subspace of $W$.
    The kernel consists of the vectors in $V$ that map to $\mathbf{0}$, and the range consists of the vectors in $W$ that get "hit" by $T$.

(2) For a linear transformation $T\colon \mathbb{R}^n \to \mathbb{R}^m$ given by $T(\mathbf{x}) = A\mathbf{x}$, the kernel is the null space of $A$ and the range is the column space of $A$. This explains why the dimension of the kernel and range of a linear transformation is called its *nullity* and *rank,* respectively.

(3) An *isomorphism* $V \to W$ of vector spaces is a linear transformation that is one-to-one (the kernel is $\{\mathbf{0}\}$) and *onto* (the range is $W$).

(4) Isomorphic vector spaces have the same dimension.

**Suggested problems.** To verify that you have understood the material, solve the following problems at the end of the section: 4–8, 14, 52.

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