Quiz 6
Math 2350
Fall 2007

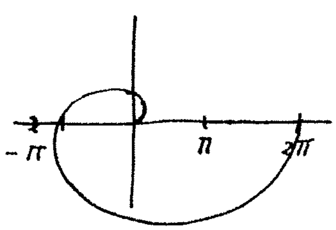
Name and section Solution

In the plane, consider the smooth curve C given by

$$\mathbf{R}(t) = \langle t\cos t, t\sin t\rangle, \quad 0 \le t \le 2\pi.$$

1. (20%) Sketch C.
2. (40%) Compute the line integral
$$\int_C \mathbf{F}\cdot \mathrm{d}\mathbf{R},$$
where $\mathbf{F} = \langle y, -x\rangle$.
3. (40%) Compute the line integral
$$\int_C \mathbf{G}\cdot \mathrm{d}\mathbf{R},$$
where $\mathbf{G} = \langle x, y\rangle$.

①



② $\bar{F}(t) = \langle t\sin t, -t\cos t\rangle$

$d\bar{R} = \langle \cos t - t\sin t, \sin t + t\cos t\rangle dt$

$$\int_C \bar{F}\cdot d\bar{R} = \int_0^{2\pi} \langle t\sin t, -t\cos t\rangle \cdot \langle \cos t - t\sin t, \sin t + t\cos t\rangle\, dt$$

$$= \int_0^{2\pi} t\cos t\sin t - t^2\sin^2 t - t\cos t\sin t - t^2\cos^2 t\, dt$$

$$= \int_0^{2\pi} -t^2(\cos^2 t + \sin^2 t)\, dt = \int_0^{2\pi} -t^2\, dt = \left| -\tfrac{1}{3}t^3 \right|_0^{2\pi} = \underline{\underline{-\frac{8\pi^3}{3}}}$$

③ $\bar{G} = \langle x, y\rangle$ is concervative with scalar potential

$g(x,y) = \frac{1}{2}x^2 + \frac{1}{2}y^2$ $\quad \bar{R}(0) = (0,0) \quad \bar{R}(2\pi) = (2\pi, 0)$

$$\int_C \bar{G}\cdot d\bar{R} = g(2\pi,0) - g(0,0) = \tfrac{1}{2}(2\pi)^2 - 0 = \underline{\underline{2\pi^2}}$$