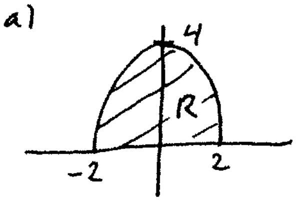
Quiz 5
Math 2350
Fall 2007

Name and section Solution

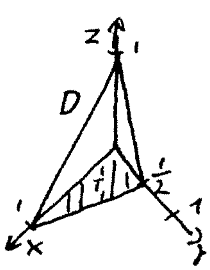
1. (50%) In the plane, consider the region R bounded by the x-axis and by the graph $y = 4 - x^2$.
   (a) Sketch R.
   (b) Compute the double integral
$$\iint_R x + 1 \, dA.$$
2. (50%) Consider the tetrahedron D bounded by the plane $x + 2y + z = 1$ and the xy-, xz-, and yz-planes.
   (a) Sketch D.
   (b) Compute the volume of D.

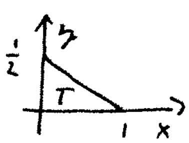
① (a)



(b)
$$\iint_R x+1\,dA = \int_{-2}^{2}\int_0^{4-x^2} x+1\,dy\,dx$$
$$= \int_{-2}^{2} \left| y(x+1) \right|_0^{4-x^2} dx = \int_{-2}^{2} (4-x^2)(x+1)\,dx$$
$$= \int_{-2}^{2} -x^3 - x^2 + 4x + 4\,dx = \left| -\tfrac{1}{4}x^4 - \tfrac{1}{3}x^3 + 2x^2 + 4x \right|_{-2}^{2}$$
$$= -4 - \tfrac{8}{3} + 8 + 8 - \left(-4 + \tfrac{8}{3} + 8 - 8\right) = 16 - \tfrac{16}{3} = \underline{\underline{\tfrac{32}{3}}}$$

②





$$Vol = \iiint_D 1\,dV = \iint_T \left(\int_0^{1-x-2y} 1\,dz \right) dA$$
$$= \iint_T 1-x-2y\,dA = \int_0^{1/2}\int_0^{1-2y} 1-x-2y\,dx\,dy$$
$$= \int_0^{1/2} \left| x - \tfrac{1}{2}x^2 - 2xy \right|_0^{1-2y} dy = \int_0^{1/2} 1-2y-\tfrac{1}{2}(1-2y)^2 - 2(1-2y)y\,dy$$
$$= \int_0^{1/2} 1 - 2y - \tfrac{1}{2} + 2y - 2y^2 - 2y + 4y^2\,dy$$
$$= \int_0^{1/2} 2y^2 - 2y + \tfrac{1}{2}\,dy = \left| \tfrac{2}{3}y^3 - y^2 + \tfrac{1}{2}y \right|_0^{1/2} = \tfrac{2}{3}\left(\tfrac{1}{8}\right) - \tfrac{1}{4} + \tfrac{1}{4} = \tfrac{1}{12}$$