

Consider the function

$$f(x, y) = x^3 - 3xy.$$

1. (20%) Find the partial derivatives  $f_x$  and  $f_y$ .
2. (10%) Show that  $P(2, 1, 2)$  is a point of the graph of  $f$ .
3. (30%) Find the equation of the tangent plane at  $P$ .
4. (10%) Find the equation of a line through  $P$  that is normal (perpendicular) to the tangent plane.
5. (30%) Let  $x = \cos t$  and  $y = \sin t$ . Find the derivative  $\frac{df}{dt}$ .

$$\textcircled{1} \quad f_x(x, y) = 3x^2 - 3y = 3(x^2 - y) \quad ; \quad f_y(x, y) = -3x$$

$$\textcircled{2} \quad f(2, 1) = 8 - 3 \cdot 2 = 2$$

$$\textcircled{3} \quad f_x(2, 1) = 3(4 - 1) = 9 \quad ; \quad f_y(2, 1) = -6$$

Tangent plane:

$$9(x - 2) + (-6)(y - 1) = 2 - 2 \quad \Leftrightarrow$$

$$9x - 6y - 10 = 0$$

$$\textcircled{4} \quad x = 2 + 9t, \quad y = 1 - 6t, \quad z = 2 - t$$

$$\textcircled{5} \quad \frac{dx}{dt} = -\sin t, \quad \frac{dy}{dt} = \cos t$$

$$\begin{aligned} \frac{df}{dt} &= 3(\cos^2 t - \sin t)(-\sin t) - 3\cos t \cos t \\ &= -3\cos^2 t(1 + \sin t) + 3\sin^2 t \end{aligned}$$