

This quiz deals with lines and planes in space.

1. (30%) Write down the equation of the line in space determined by the point $P_1 = (2, -6, 5)$ and the direction vector $\mathbf{a} = \langle -1, 3, 2 \rangle$.
2. (30%) Write down the equation of the plane in space determined by the point $P_1 = (4, 1, -2)$ and the normal vector $\mathbf{a} = \langle 1, 1, -2 \rangle$.
3. (25%) Consider the equation $2x - y + 3z = 7$. Does it determine a line or a plane? Find a point that lies on this line/plane.
4. (15%) Find the intersections points of the two planes given by:
 - The point $P_1 = (2, -6, 5)$ and the normal vector $\mathbf{a}_1 = \langle -1, 3, 2 \rangle$
 - The point $P_2 = (5, 4, 1)$ and the normal vector $\mathbf{a}_2 = \langle 2, -6, -4 \rangle$

$$\textcircled{1} \quad \begin{aligned} x &= 2 - t \\ y &= -6 + 3t \\ z &= 5 + 2t \end{aligned} \quad \text{or} \quad \frac{x-2}{-1} = \frac{y+6}{3} = \frac{z-5}{2}$$

$$\textcircled{2} \quad (x-4) + (y-1) - 2(z+2) = 0 \quad \text{or} \quad x + y - 2z - 9 = 0$$

$\textcircled{3}$ It is a plane. The point $(0, -7, 0)$ is on this plane.

$\textcircled{4}$ The two planes are parallel as $\bar{\mathbf{a}}_2 = -2\bar{\mathbf{a}}_1$.
The equation of the first plane

$$-(x-2) + 3(y+6) + 5(z-2) = 0$$

is not satisfied by P_2 :

$$-(5-2) + 3(4+6) + 5(1-2) = -3 + 30 - 5 = 22$$

Thus the two planes are different and parallel so they have no intersection points.