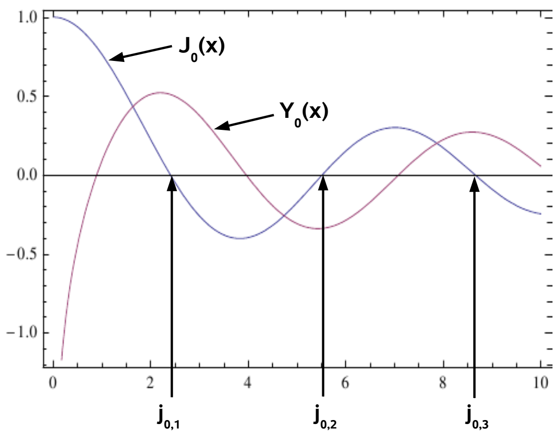
Math 3351 - Fall 2009

Quiz #9

The picture shows the Bessel functions of order zero, $J_0(x)$ and $Y_0(x)$



1. **Label the picture to indicate clearly which curve is $J_0(x)$ and which is $Y_0(x)$**

   As shown in the labeled picture, J_0 is the blue curve, Y_0 the red curve. The most obvious difference is that Y_0 blows up as $x \to 0$.

2. **Referring to the picture, read off approximate values for $j_{0,1}$, $j_{0,2}$, and $j_{0,3}$. Use arrows and labels to indicate clearly which features of the graph you used to find these values.**

   The numbers $j_{n,m}$ are the $m-$th roots of the Bessel function $J_n(x)$. In other words, $J_n(j_{n,m}) = 0$. From the figure, we can read approximate values: $j_{0,1} \approx 2.4$, $j_{0,2} \approx 5.5$, and $j_{0,3} \approx 8.6$. More accurate values can be computed by Mathematica,

   ```
   In[2]:= Table[BesselJZero[0, m], {m, 1, 3}] // N

   Out[2]= {2.40483, 5.52008, 8.65373}
   ```

3. **Given a function $R(r) = AJ_0(kr) + BY_0(kr)$ and boundary conditions $|R(0)| < \infty$, $R(a) = 0$, find:**

   (a) **The three lowest values of k for which nontrivial solutions exist**

   The function $Y_0(x)$ blows up at zero, so the boundary condition $|R(0)| < \infty$ requires that $B = 0$. Therefore the solution is $R(r) = AJ_0(kr)$. The boundary condition $AJ_0(ka) = 0$ requires either $A = 0$ (trivial solution) or $J_0(ka) = 0$. Therefore nontrivial solutions will exist only when $ka = j_{0,n}$. The lowest three nontrivial k will be $k_1 \approx 2.4/a$, $k_2 \approx 5.5/a$, and $k_3 \approx 8.6/a$.

   (b) **Whichever of A or B can be determined with the information given**

   As explained in part (a), we know $B = 0$. The other coefficient, A, cannot be determined by the information given.