Sample Computational Project
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Math 3350, Section $\sqrt{-1}$
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This is an example of what a successful computational project report should look like. It must be formatted using your favorite word processing program (such as Microsoft Word on Windows or Mac, or OpenOffice on Linux). Figures, equations, and tables must be included as part of the document. You should be able to type equations neatly using a program such as the Word Equation Editor.

Problem 1: Solve $y'(x)=y$ with initial conditions $y(0)=1$, on the interval [0,2].

I will solve this problem using Euler's method, and compare the Euler approximations with the exact solution. I will then look at the behavior of the error as the number of steps changes, and use these results to check the theoretical prediction that the global error in an Euler approximation varies as $O(h)$.

1.1: Calculation of exact solution.

The equation $y'(x)=y$ is separable, so I will use the method of separation of variables. The first step is to separate variables and integrate both sides. The lower limits of integration are set from the initial conditions; the upper limits are the variables y and x.

$$\int_1^y \frac{dy}{y} = \int_0^x dx$$

Doing the integration and evaluating at the limits of integration results in

$$\log y = x$$

from which I can solve for y, giving the solution

$$y(x) = e^x.$$

1.2: Verification of exact solution

An essential step is to check the correctness of the exact solution I have calculated above.

- I first check that the solution $y(x)=e^x$ satisfies the initial conditions. Plugging in $x=0$, I find $y(0)=e^0=1$ so I recover the correct initial value $y(0)=1$.
- Next, I check that the solution satisfies the differential equation. The derivative of $y(x)=e^x$ is $y'(x)=e^x$ so $y'=y$ as it should.
- Having checked that the solution satisfies both the initial conditions and the differential equation, I have verified that I've solved the initial value problem correctly.

1.3: Matlab function for right-hand side of $y'=y$

The Matlab function to evaluate the right-hand side of $y'=y$ is shown below. As required, there is a comment line with the student's name in it. The Matlab file directly into this document. When including computer code in a document it's conventional to use a fixed-width font such as Courier.

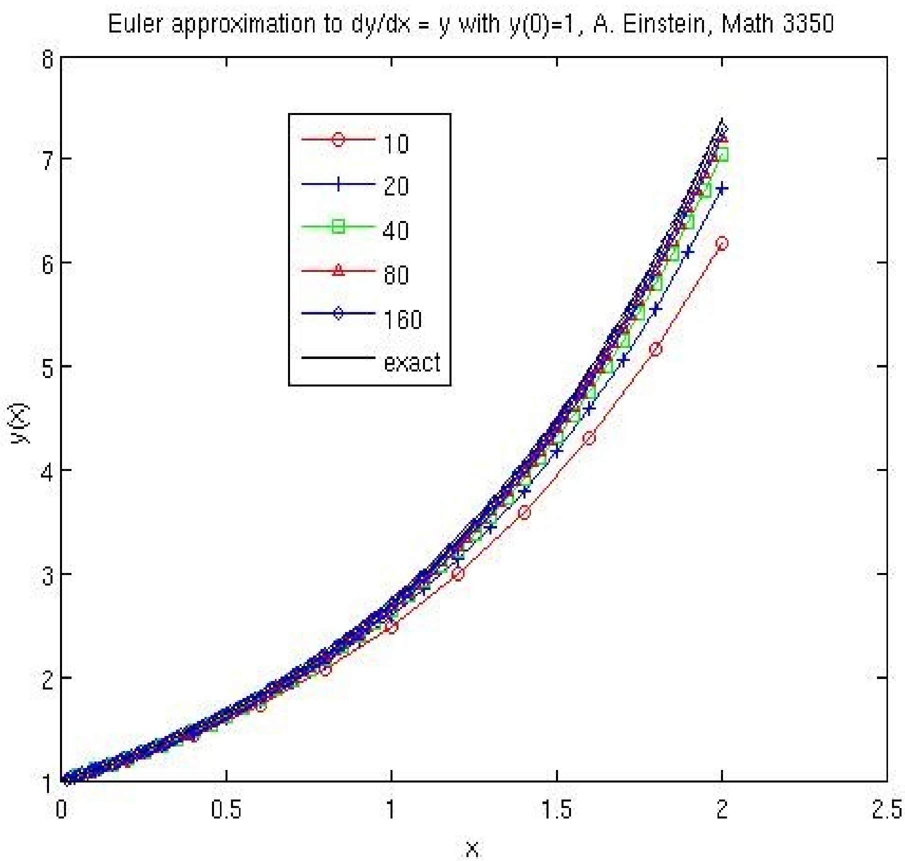
```
function rtn = simpleFunc(x, y, modelData)
% By Albert Einstein, Math 3350
% simpleFunc() is used to define the differential equation
%    dy/dx = alpha * y
%
```

```
% Input arguments:
% x          -- independent variable
% y          -- dependent variable
% modelData  -- model-specific data. The coefficient alpha is passed
%               as modelData(1).
% Output:
% rtn        -- the value of dy/dx at the evaluation point.

    alpha = modelData(1);
    rtn = alpha * y;
```

1 1.4: Numerical results (Euler's Method)

In Figure 2 I plot the exact solution of problem 1 and several Euler approximations using 10, 20, 40, and 80 steps. The calculations are done on the interval [0,2]. The runs with various step sizes are distinguished by symbols as indicated in the legend. As the number of steps increases (and thus the step size decreases) the curves are approaching the exact solution. I next measure the rate at which they approach the exact solution, as described by the order of accuracy.



In class we saw that the global error (that is, the error after taking multiple steps to reach a fixed end point) in Euler's method is proportional to the step size h. To test that, I have measured the error

$$E = |y_{exact} - y_{Euler}|$$

at the stopping point x=2. Assuming the global error varies as $O(h^p)$ where h is the step size, I can estimate p by comparing the errors at different values of h. If E_1 is the error at step size h_1 and E_2 the error at step size h_2, then the errors at the two steps should be related by the equation

$$\frac{E_1}{E_2}=\left(\frac{h_1}{h_2}\right)^p$$

and therefore if I've measured E_1 and E_2 I can solve for p:

$$p=\frac{\log\left(\frac{E_1}{E_2}\right)}{\log\left(\frac{h_1}{h_2}\right)}$$

The exponent p is called the order of accuracy of the method, and for Euler's method I expect it to be $p=1$. Error results for the different runs are summarized in Table 1 below. The first column contains the number of steps for the different runs. The second column shows the errors in the approximate solutions computed at the end point $x=2$ for the step sizes used. The third column lists the estimates of the order of accuracy, p, computed by comparing the error with N steps to the error with N/2 steps.

Table 1

N steps	Error at x=2	Estimate of p
10	1.197	---
20	0.662	0.845
40	0.349	0.924
80	0.180	0.959
160	0.091	0.980

The order of accuracy p is seen to be approaching 1 as the number of steps increases, as predicted theoretically for Euler's method.