Math 3350, Sample Test # 2, Name ____________________

1. Give the interval of existence of the unique solution to the initial value problem

$$\frac{(x^2-36)}{(x+2)}y'''' + (x^2-1)y'' - x^2y' + \frac{x}{x+4}y = \frac{x^2}{x^2+1},$$

with ICs $y(2) = 3,\ y'(2) = -2,\ y''(2) = 4,\ y'''(2) = 0.$

ANSWER: $(x^2-36) = 0$ at $x = \pm 6$, discontinuities at $x = -2, x = -4$ initial point $x_0 = 2, \Rightarrow\ -2 < x < 6$

2. Solve the initial value problem $y'' + y' - 2y = 0\ \ y(0) = 4,\ \ y'(0) = 1.$

ANSWER: $\boxed{y = 3e^x + e^{-2x}}$

3. Find the general solution $y'' - 2y' + 10y = 0.$

ANSWER: $\boxed{y = c_1e^x\cos(3x) + c_2e^x\sin(3x))}$

4. Find the general solution of $y^{(4)} - 5y'' - 36y = 0.$

ANSWER: $\boxed{y = c_1\cos(2x) + c_2\sin(2x) + c_3e^{3x} + c_4e^{-3x}}$

5. Find the general solution $y(x)$ of $y''' + y'' + y' + y = 0.$

ANSWER: $\boxed{y = c_1e^{-x} + c_2\cos(x) + c_3\sin(x)}$

6. Find a **candidate** of a particular solution $y_p(x)$ (DO NOT SOLVE FOR CONSTANTS)

$y'' + 4y' + 4y = 2xe^{-2x} + 8x\sin(2x).$

ANSWER: $\boxed{y = x^2(Ax+B)e^{-2x} + (Cx+D)\sin(2x) + (Ex+F)\cos(2x)}$

7. Use *undetermined coefficients* to find a particular solution for $y'' - 3y' + 2y = 10\sin(x)$. Also find the general solution.

ANSWER: $\boxed{y = c_1e^x + c_2e^{2x} + 3\cos(x) + \sin(x)}$

8. Use *variation of parameters* to find a particular solution $y'' - 2y' + y = 4e^x$. Also find the general solution.

ANSWER: $\boxed{y = c_1e^x + c_2xe^x + 2x^2e^x}$

9. Solve the initial value problem $x^2y'' + 3xy' + y = 0$ with $y(1) = 2,\ y'(1) = -1.$

ANSWER: general solution $y = c_1x^{-1} + c_2\ln(x)x^{-1}$ and $y = 2x^{-1} + \ln(x)x^{-1}$