Math 3350, Sample Test # 1, Name ____________________________

IGS means Implicit General Solution

1. Given the IVP $(x^2-4)y' + x^2 y = \dfrac{x}{x-7}$ with $y(3) = 1$. On what interval does the fundamental existence theory for first order initial value problems guarantee there is a unique solution.

   ANSWER: $\boxed{2 < x < 7}$

2. Given the autonomous equation $y' = y^2(y^2-1)(y-2)$,

   (a) Find and classify all critical points as:
   AS for *asymptotically stable*, **US** for *unstable*, or **SS** for *semi-stable*.

   (b) If a solution $y(x)$ has initial condition $y(0) = 3/2$ find $\lim_{x\to\infty} y(x)$

   ANSWER: $\boxed{y=-1, US, \quad y=0, SS, \quad y=1, AS, \quad y=2, US \text{ and } \lim_{x\to\infty} y(x) = 1}$

3. Separable Solve the initial value problem $(x^2+1)y' = 2x\cos^2(y)$ with $y(0) = \dfrac{\pi}{4}$. Find an explicit solution.

   ANSWER: $\boxed{y(x) = \arctan\left(1 + \ln\left(x^2+1\right)\right)}$

4. Separable Solve the initial value problem $y' = 3x\sqrt{x^2+1}\sec(y)$ with $y(0) = 0$. Find an explicit solution.

   ANSWER: $\boxed{y = \sin^{-1}\left((1+x^2)^{3/2} - 1\right)}$

5. Solve the First Order Linear IVP $xy' + y = 3x^2$, $y(1) = 2$. Find an explicit solution.

   ANSWER: $\boxed{y = x^2 + \dfrac{1}{x}}$

6. Solve the Exact equation $(\sin(y) - y\sin(x))\,dx + (1 + x\cos(y) + \cos(x))\,dy = 0$.

   ANSWER: $\boxed{x\sin(y) + y\cos(x) + y = C}$

7. Find an integrating factor and show that it is one (do not solve) $2x\tan(y)\,dx + \sec^2(y)\,dy = 0$.

   ANSWER: $\boxed{\mu = e^{x^2}, \Rightarrow \ M = 2xe^{x^2}\tan(y),\ N = e^{x^2}\sec^2(y),\ M_y = 2xe^{x^2}\sec^2(y) = N_x}$

8. Find an IGS for the Homogeneous equation $y' = \dfrac{y^2+xy}{x^2}$.

   ANSWER: $\boxed{xy^{-1} + \ln(x) = C}$

9. Use the substitution $z = (x-y)$ to find an IGS for $y' = (x-y)^2 + 1$.

   ANSWER: $\boxed{\dfrac{1}{(x-y)} = x + C}$

10. Find an IGS for the Bernoulli equation $y' - \dfrac{1}{x}y = \dfrac{4}{(xy)^2}$.

    ANSWER: $\boxed{y^3 + \dfrac{3}{x} - Cx^3 = 0}$