Supplemental Laplace Transform Problems

Find the Laplace and inverse Laplace transform:

1. $e^{-5t}\sin(2t) \quad \Rightarrow \qquad 2\left(s^2+10\,s+29\right)^{-1}$
2. $t^2e^{-5t} \quad \Rightarrow \qquad 2\,(s+5)^{-3}$
3. $(e^{2t}-e^{-2t})^2 \quad \Rightarrow \qquad \frac{1}{s-4}+\frac{1}{s+4}-\frac{2}{s}$
4. $t^3+e^{2t}\cos(3t) \quad \Rightarrow \qquad 6\,s^{-4}+\frac{s-2}{(s-2)^2+9}$
5. $\frac{s}{s^2+2s+2} \quad \Rightarrow \qquad e^{-t}\left(\cos(t)-\sin(t)\right)$
6. $\frac{2s}{s^2+2s+5} \quad \Rightarrow \qquad e^{-t}\left(2\,\cos(2\,t)-\sin(2\,t)\right)$
7. $\frac{(s+2)}{s^2+2s+1} \quad \Rightarrow \qquad (t+1)\,e^{-t}$
8. $\frac{2(s+1)}{s^2+2s} \quad \Rightarrow \qquad 1+e^{-2\,t}$

Find the partial fraction expansion:

1. $\frac{2s}{s^2-4} \quad \Rightarrow \qquad (s-2)^{-1}+(s+2)^{-1}$
2. $\frac{(3s+1)}{s^2+4s} \quad \Rightarrow \qquad 1/4\,s^{-1}+11/4\,(s+4)^{-1}$
3. $\frac{3s^2-s+2}{s(s^2+1)} \quad \Rightarrow \qquad 2/s+(s-1)/(s^2+1)$
4. $\frac{(s^2+4s+4)}{s^2(s^2+4)} \quad \Rightarrow \qquad 1/s+1/s^2-s/(s^2+4)$
5. $\frac{(s+1)}{s(s-1)(s^2+1)} \quad \Rightarrow \qquad -1/s+1/(s-1)-1/(s^2+1)$
6. $\frac{1}{(s-2)^2} \quad \Rightarrow \qquad \frac{1}{(s-2)^2}$

Find the Laplace and inverse Laplace transform with Heaviside function:

1. $u(t-1)e^{2t} \Rightarrow \quad \frac{e^{-s+2}}{s-2}$
2. $u(t-1)t \Rightarrow \quad \frac{e^{-s}(s+1)}{s^2}$
3. $u(t-\pi)\sin(t) \Rightarrow \quad -\frac{e^{-s\pi}}{s^2+1}$
4. $u(t-2)t^2 \Rightarrow \quad 2\frac{e^{-2s}(2s^2+2s+1)}{s^3}$
5. $u(t-2)(2t-1) \Rightarrow \quad \frac{e^{-2s}(3s+2)}{s^2}$
6. $u(t-1)e^{2t}t \Rightarrow \quad \frac{e^{-s+2}(s-1)}{(s-2)^2}$
7. $\frac{se^{-5s}}{s^2+4} \Rightarrow \quad u(t-5)\cos(2t-10)$
8. $\frac{e^{-s}}{s^2+s} \Rightarrow \quad u(t-1)\left(1-e^{-t+1}\right)$

Solve the initial value problem:

1. $y''-2y'+y=t,\ \ y(0)=0,\ \ y'(0)=0 \Rightarrow \quad y(t)=(t-2)e^t+2+t$
2. $y''-y=2u(t-1),\ \ y(0)=0,\ \ y'(0)=0 \Rightarrow \quad y(t)=u(t-1)\left(-2+e^{t-1}+e^{-t+1}\right)$
3. $y''+4y=-4u(t-\pi),\ \ y(0)=0,\ y'(0)=0 \Rightarrow \quad y(t)=u(t-\pi)\left[\cos(2t)-1\right]$
4. $y''+4y=2\delta(t-\pi),\ \ y(0)=0,\ \ y'(0)=2 \Rightarrow \quad y(t)=\sin(2t)+u(t-\pi)\sin(2t)$
5. $y''-y'=\delta(t-1)t,\ \ y(0)=1,\ \ y'(0)=0 \Rightarrow \quad y(t)=1+u(t-1)\left[e^{t-1}-1\right]$

Solve the integral equation:

1. $y(t)=6+\int_0^t y(t-\tau)\,e^{-3\tau}\,d\tau, \quad \Rightarrow \quad y(t)=9-3e^{-2t}$
2. $y(t)=2e^{-t}+\int_0^t y(t-\tau)\,d\tau, \quad \Rightarrow \quad y(t)=e^t+e^{-t}$
3. $y(t)=-2t+\int_0^t y(t-\tau)\,\tau\,d\tau, \quad \Rightarrow \quad y(t)=e^{-t}-e^t$
4. $y(t)=\cos(t)+2\int_0^t y(t-\tau)\,e^{-2\tau}\,d\tau, \quad \Rightarrow \quad y(t)=\cos(t)+2\sin(t)$