Supplemental Second Order Linear Non-Homogeneous Equations

In the following find a candidate for a particular solution

6. $y'' - 4y = 4e^{2x} \quad \Rightarrow \quad y(t) = c_1 e^{2x} + c_2 e^{-2x} + xe^{2x}$
7. $y'' - 2y' + 2y = 2e^x \cos(x) \quad \Rightarrow \quad y_p = Axe^x \sin(x) + Bxe^x \cos(t)$
8. $y'' - 2y' + y = 2e^x \quad \Rightarrow \quad y_p = Ax^2 e^x$
9. $y'' - 4y' + 3y = x^2 + \sin(x) \quad \Rightarrow \quad y_p = (Ax^2 + Bx + C) + (D\sin(x) + E\cos(x))$
10. $y'' - y' - 2y = e^x + x \quad \Rightarrow \quad y_p = Ae^x + Bx + C$
11. $y'' + 9y = \sin(2x) \quad \Rightarrow \quad y_p = A\sin(2x) + B\cos(2x)$
12. $y'' - 3y' + 2y = e^x \quad \Rightarrow \quad y_p = Axe^x$
13. $y'' - y' = x + 1 \quad \Rightarrow \quad y_p = Ax^2 + Bx$

Find the general solution, i.e., find y_c and a y_p and write $y = y_c + y_p$

1. $y'' - y' = 4x \quad \Rightarrow \quad y = c_1 e^x + c_2 - 4x - 2x^2$
2. $y'' - 2y' + y = x + e^{2x} \quad \Rightarrow \quad y = c_1 e^x + c_2 xe^x + x + 2 + e^{2x}$
3. $y''' - y'' = 4 - 2\cos(x) \quad \Rightarrow \quad y = \sin(x) - \cos(x) - 2x^2 + Ae^x + Bx + C$

Solve

1. $y'' - y' - 2y = 3e^{2x}$ with $y(0) = 0$ and $y'(0) = -2$ $\qquad y(x) = e^{-x} - e^{2x} + xe^{2x}$
2. $y'' - y = x$ with $y(2) = e^2 - 2$ and $y'(2) = e^2 - 1$ $\qquad y(x) = e^x - x$
3. $y'' + y' - 2y = 14 + 2x - 2x^2$ with $y(0) = 0$ and $y'(0) = 0$ $\qquad y(x) = 2e^{-2x} + 4e^x - 6 + x^2$
4. $y'' + y = 2e^x$ with $y(0) = 0$ and $y'(0) = 0$ $\qquad y(x) = -\sin(x) - \cos(x) + e^x$
5. $y'' + y = 2\sin(x)$ with $y(0) = 0$ and $y'(0) = 0$ $\qquad y(x) = \sin(x) - x\cos(x)$