Supplemental First Order Problems

Homogeneous

1. $2xyy' - y^2 + x^2 = 0$ **ANS:** $x^2 + y^2 = cx$
2. $xy' = (x + y)$ **ANS:** $y = (\ln(x) + c)\, x$
3. $y' = \dfrac{x - y}{x + y}$ **ANS:** $y^2 + 2xy - x^2 = c$
4. $x^2 y' = y^2 + xy + x^2$ **ANS:** $y = \tan(\ln(x) + c)x$
5. $y' = \dfrac{2y^4 + x^4}{xy^3}$ **ANS:** $y^4 = cx^8 - x^4$
6. $y' = \dfrac{x^2 + y^2}{xy}$ **ANS:** $y^2 = x^2 \ln(x^2) + cx^2$

Substitution

1. $y' = (y - x)^2$ set $v = y - x$ **ANS:** $-\ln(y - x - 1) + \ln(y - x + 1) + 2\,x = c$
2. $y' = \tan(x + y) - 1$ set $v = x + y$ **ANS:** $x - \ln(\sin(x + y)) = c$
3. $y' = \dfrac{y - x + 1}{y - x + 2}$ set $v = y - x$ **ANS:** $-x + 1/2\ (y - x)^2 + 2\,y = c$

Bernoulli

1. $y' + xy = xy^{-1}$ **ANS:** $y^2 - 1 - e^{-x^2} c = 0$
2. $y' + x^{-1} y = x^{-1} y^2$ **ANS:** $y^{-1} - 1 - cx = 0$
3. $y' + y = xy^{-1}$ **ANS:** $y^2 + 1/2 - x - e^{-2\,x} c = 0$
4. $y' - \dfrac{3}{x} y = x^4 y^{1/3}$ **ANS:** $y^{2/3} = cx^2 + \dfrac{2}{9} x^5$

Exact

1. $2xy\,dx + (1 + x^2)\,dy = 0$ **ANS:** $x^2 y + y = c$
2. $(x + \sin(y))\,dx + (x\cos(y) - 2y)\,dy = 0$ **ANS:** $\dfrac{x^2}{2} + x\sin(y) - y^2 = c$
3. $(y + 2xy^3)\,dx + (1 + 3x^2 y^2 + x)\,dy = 0$ **ANS:** $xy + x^2 y^3 + y = c$