Formulas Cullen Zill AEM Chapters 1 and 2

I. (RHS only contains x) $\boxed{y' = f(x)}$ $\Rightarrow$ $\boxed{y = \int f(x)\,dx + C}$

II. (Separable) $\boxed{f(y)y' = g(x)}$ $\Rightarrow$ $\boxed{\int f(y)\,dy - \int g(x)\,dx = C}$

III. (First Order Linear) $\boxed{y' + P(x)y = Q(x)}$ Set $\boxed{\mu = e^{\int P dx}}$ then $\boxed{[y\mu]' = \mu Q}$

IV. (Exact) $\boxed{M(x,y)\,dx + N(x,y)\,dy = 0}$ is exact if $\boxed{\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}}$. If exact then there exists $F(x,y)$ so that $\boxed{M = \dfrac{\partial F}{\partial x}, \quad N = \dfrac{\partial F}{\partial y}}$. Use these to find solution $\boxed{F(x,y) = C}$.

V. (Integrating factor) $\boxed{\widetilde{M}(x,y)\,dx + \widetilde{N}(x,y)\,dy = 0}$ $\boxed{f(x) = \dfrac{\widetilde{M}_y - \widetilde{N}_x}{\widetilde{N}} \quad \Rightarrow \quad \mu = e^{\int f(x)\,dx}}$

VI. (Integrating factor) $\boxed{\widetilde{M}(x,y)\,dx + \widetilde{N}(x,y)\,dy = 0}$ $\boxed{g(y) = \dfrac{\widetilde{M}_y - \widetilde{N}_x}{\widetilde{M}} \quad \Rightarrow \quad \mu = e^{-\int g(y)\,dy}}$

VII. (Substitution: RHS Linear in x and y) $\boxed{y' = f(ax + by + c)}$ the substitution $\boxed{v = ax + by + c}$ transforms the problem to $\boxed{v' = a + bf(v)}$ which is separable.

VIII. (Substitution: Homogenous) $\boxed{y' = f(y/x)}$ the substitution $\boxed{v = y/x}$ produces $\boxed{xv' + v = f(v)}$ which is separable.

IX. (Substitution: Bernoulli) $\boxed{y' + P(x)y = Q(x)y^n, n \neq 0, 1}$ then the substitution $\boxed{v = y^{1-n}}$ provides a first order linear equation for v $\boxed{v' + (1-n)P(x)v = (1-n)Q(x)}$ which is First Order Linear.