Supplemental Second Order Euler-Cauchy Equations

1. $x^2y'' + xy' - 9y = 0$ **ANS:** $y(x) = c_1\, x^3 + \dfrac{c_2}{x^3}$

2. $x^2y'' + 3xy' + 10y = 0$ **ANS:** $y(x) = \dfrac{c_1\, \sin(3\, \ln(x))}{x} + \dfrac{c_2\, \cos(3\, \ln(x))}{x}$

3. $x^2y'' + 3xy' + y = 0$ **ANS:** $y(x) = \dfrac{c_1}{x} + \dfrac{c_2\, \ln(x)}{x}$

4. $x^2y'' + 2xy' - 6y = 0$ **ANS:** $y(x) = c_1\, x^2 + \dfrac{c_2}{x^3}$

5. $y'' + \dfrac{6}{x}y' + \dfrac{4}{x^2}y = 0$ **ANS:** $y(x) = \dfrac{c_1}{x^4} + \dfrac{c_2}{x}$

6. $9x^2y'' + 15xy' + y = 0$ **ANS:** $y(x) = \dfrac{c_1}{\sqrt[3]{x}} + \dfrac{c_2\, \ln(x)}{\sqrt[3]{x}}$

7. $y'' - \dfrac{1}{x}y' + \dfrac{5}{x^2}y = 0$ **ANS:** $y(x) = c_1\, x \sin(2\, \ln(x)) + c_2\, x \cos(2\, \ln(x))$

8. $x^2y'' + 9xy' + 17y = 0$ **ANS:** $y(x) = \dfrac{c_1\, \sin(\ln(x))}{x^4} + \dfrac{c_2\, \cos(\ln(x))}{x^4}$

9. $x^2y'' - 4xy' + 4y = 0, \quad y(1) = -2, \;\; y'(1) = -11$ **ANS:** $y(x) = -3\, x^4 + x$

10. $x^2y'' - xy' + 2y = 0, \quad y(1) = -1, \;\; y'(1) = -11$ **ANS:** $y(x) = -x \cos(\ln(x)) - 10x \sin(\ln(x))$

11. $x^2y'' + 5xy' + 8y = 0, \quad y(1) = 5, \;\; y'(1) = 18$ **ANS:** $y(x) = 14\, \dfrac{\sin(2\, \ln(x))}{x^2} + 5\, \dfrac{\cos(2\, \ln(x))}{x^2}$

Find the general solution (note you must use variation of parameters to find y_p)

12. $x^2y'' - 4xy' + 6y = 42x^{-4}$ **ANS:** $c_1x^2 + c_2x^3 + x^{-4}$

13. $x^2y'' - 2xy' + 2y = 5x^3 \cos(x)$ **ANS:** $c_1x + c_2x^2 - 5x\cos(x)$

14. $xy'' - y' = x^2e^x$ **ANS:** $c_1 + c_2x + (x+1)e^x$

15. $x^2y'' - 2xy' + 2y = 24x^{-2}$ **ANS:** $c_1x + c_2x^2 + 2x^{-2}$