Control and Optimization of Energy Efficient Buildings: A Control Grand Challenge

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Interdisciplinary Center for Applied Mathematics

Workshop in Celebration of the Life, Mathematics and Memories of Christopher I. Byrnes

September 10 - 12, 2010

Air Flow in a Hospital Suite
Christopher I. Byrnes

ALWAYS HAD A SMILE
ENJOYED LIFE
LOVED HIS WIFE
LOVED MATHEMATICS

CHRIS HAD "THAT VISION THING" ... THE LAST TIME WE TALKED
IT WAS ABOUT DESIGN & CONTROL OF BUILDINGS
Energy and Buildings

WHY BUILDINGS
Energy and Buildings

WHY BUILDINGS

AND NOT THIS
Energy and Buildings

WHY BUILDINGS

AND NOT THIS
Impact of Energy Efficient Buildings

HUGE

• A 50 percent reduction in buildings’ energy usage would be equivalent to taking every passenger vehicle and small truck in the United States off the road.

• A 70 percent reduction in buildings’ energy usage is equivalent to eliminating the entire energy consumption of the U.S. transportation sector.
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DESIGN, CONTROL AND OPTIMIZATION OF WHOLE BUILDING SYSTEMS IS THE ONLY WAY TO GET THERE

? WHY ?
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**Figure 3.7: Life cycle energy use**

- 84% of energy consumed in buildings is during the use of the building
- 4% is during maintenance and renovation
- 12% is during manufacturing, transport, and construction
- 84% (heating, ventilation, hot water & electricity)
Impact of Energy Efficient Buildings

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REQUIRES COMBINING - MODELING, COMPLEX MULTISCALE DYNAMICS, CONTROL, OPTIMIZATION, SENSITIVITY, HIGH PERFORMANCE COMPUTING … THINGS THAT WE DO!
Existence Proof

**Energy Retrofit**
- 10-30% Reduction

**Very Low Energy**
- >50% Reduction

**Cityfront Sheraton**
- Chicago IL
- 1.2M ft², 300 kWhr/m²
- 5753 HDD, 3391 CDD
- VS chiller, VFD fans, VFD pumps
- Condensing boilers & DHW

**Bonn Germany**
- 1M ft², 75 kWhr/m²
- 6331 HDD, 1820 CDD
- No fans or Ducts
- Slab cooling
- Façade preheat
- Night cool

**Tulane Lavin Bernie**
- New Orleans LA
- 150K ft², 150 kWhr/m²
- 1513 HDD, 6910 CDD
- Porous Radiant Ceiling, Humidity Control Zoning, Efficient Lighting, Shading

**LEED Design**
- 20-50% Reduction

Courtesy of Clas Jacobson UTC
Whole Building Systems

Building Operating Conditions
- Fire / Smoke Detection and Alarm
- Video
- Facility access

Cost Utilities
- Building Geometry
- Building Insulation

Weather
- IT Network
- Building Management System
- Safety & Security

Envelop Structure
- Information Management

Loads
- Office Equipment
- Water Heating
- Other Loads

Lighting
- Motion Sensors
- Lights & Fixtures

Heating, Ventilation, Air Conditioning
- Thermostat
- Heating & AC Equipment
- Distribution (Fans, Pumps)

Electrical
- On-Site Gen
- Grid

Information Thermal Power

Link that is not always exploited
Multi-Scale in Time and Space

- **Length scales**
  - Building-scale: $O(10^2-10^3 \text{m})$
  - Floor-scale: $O(10^2 \text{m})$
  - Room-scale: $O(1 \text{m})$

- **Time scales**
  - $O(\mu\text{sec})$
  - $O(\text{sec})$
  - $O(1-10 \text{minutes})$
  - $O(1 \text{hr})$

**Systems**
- Centralized actuation (louver/damper)
- Occupant movement/egress (walk/elevator)
- Air Handling Unit (HVAC+ducts)
- Active filtration
- Temperature, Pressure, CO$_2$, Threat Sensors
- Distributed actuation
- Distributed filtration

**Active filtration components**
Whole Buildings Are Complex System

A whole building system is a complex system because:

1. The system components do not necessarily have mathematically similar structures and may involve different scales in time or space;
2. The number of components may be large, sometimes enormous;
3. Components can be connected in a variety of different ways, most often nonlinearly and/or via a network. Furthermore, local and system wide phenomena may depend on each other in complicated ways;
4. The behavior of the overall system can be difficult to predict from the behavior of individual components. Moreover, the overall system behavior may evolve along qualitatively different pathways that may display great sensitivity to small perturbations at any stage.

In addition to the above definition, even a single room can be a complex system if one is concerned with multi-physics dynamics such as coupled (chemically reacting) air flows, thermal profiles, energy flows, air quality, room geometry and the supervisory (closed-loop, possible human in the loop) control.
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MATHEMATICAL DEFINITION OF A COMPLEX SYSTEM

The general technical challenge was issued by the U.S. Secretary of Energy, Dr. Steven Chu:

“We need to do more transformational research at DOE … including computer design tools for commercial and residential buildings that enable reductions in energy consumption of up to 80 percent with investments that will pay for themselves in less than 10 years.”

Secretary of Energy Dr. Steven Chu, House Science Committee Testimony, March 17, 2009
Technical Challenges

**Time scale separation in current building systems and controls**

- **length scales**
  - building-scale \(O(10^2-10^3 \text{m})\)
  - floor-scale \(O(10^2 \text{m})\)
  - room-scale \(O(1 \text{m})\)

- **Fundamentals**
  - Conventional Communication
  - Mechanical Ventilation Heating Cooling Electric Lighting
  - Building Envelope Thermals

- **Occupancy**

**Tightly coupled dynamics over broad range of spatial scales for very low energy buildings**

- **length scales**
  - building-scale \(O(10^2-10^3 \text{m})\)
  - floor-scale \(O(10^2 \text{m})\)
  - room-scale \(O(1 \text{m})\)

- **Fundamentals**
  - Low Actuation Computation/Processing/Controls & Wireless Communication
  - Passive Ventilation Heating Cooling Daylighting
  - Building Envelope Thermals

**From 30% efficiency to 70-80% efficiency**
Technical Challenges

- **MODELING AND SIMULATION TOOLS** THAT CAPTURE THE CORRECT BUILDING PHYSICS AT ALL KEY TIME AND SPATIAL SCALES ARE ESSENTIAL
- **HOWEVER, BUILDING SIMULATION ALONE IS NOT SUFFICIENT**
- **MATHEMATICAL AND COMPUTATIONAL TOOLS MUST BE DEVELOPED SPECIFICALLY TO ALLOW INTEGRATION AND TO INTERFACE WITH:**
  - DESIGN – CONTROL – OPTIMIZATION – SENSITIVITY – UNCERTAINTY TOOLS
- **ALSO, REVOLUTIONARY ADVANCES WILL OCCUR ONLY IF THE ALGORITHMS AND COMPUTATIONAL TOOLS ...**
  - ENABLE “PLUG-AND-PLAY” FOR NEW COMPONENT TECHNOLOGIES
  - ARE BASED ON STATE OF THE ART COMPUTATIONAL SCIENCE
  - HAVE USER INTERFACES SUITABLE FOR BROAD BUILDING STOCKS
  - BE BUILT ON OPEN SOURCE SOFTWARE
  - TAKE ADVANTAGE OF MODERN COMPUTER AND COMPUTER SCIENCE TECHNOLOGY INCLUDING HIGH PERFORMANCE COMPUTING
BARRIERS

1. EXISTING BUILDING SIMULATION AND ENERGY TOOLS ARE NOT SUITABLE FOR DESIGN, OPTIMIZATION AND CONTROL OF WHOLE BUILDING SYSTEMS
   - THEY WERE NOT DESIGNED FOR THESE PURPOSES
   - THEY WERE DESIGNED TO RUN ON PC TYPE PLATFORMS AND HENCE FORCED TO USE CRUDE MODELS OF THE PHYSICS - CHEMISTRY
   - DO NOT TAKE ADVANTAGE OF STATE-OF-THE ART ALGORITHMS AND NEW COMPUTER PLATFORMS
   - OFTEN IGNORE SPATIAL FEATURES AND TIME SCALES THAT ARE ESSENTIAL TO OPTIMIZATION AND DESIGN TOOLS

2. EXISTING MODELS DO NOT ADDRESS UNCERTAINTY, MULTI-SCALE DYNAMICS AND REAL-TIME REQUIREMENTS

3. EXISTING SOFTWARE IS OFTEN BASED ON CRUDE MODELS, DOES NOT ALLOW FOR EASY INTEGRATION OF DESIGN AND CONTROL TOOLBOXES AND GUI’s / USER INTERFACES DO NOT EXIST – “USED ONLY BECAUSE IT IS FREE”

MISCONCEPTIONS

- IF I CAN SIMULATE A SYSTEM, THEN I CAN OPTIMIZE OR CONTROL IT or I MUST BE ABLE TO CONDUCT HIGH FIDELITY SIMULATIONS BEFORE I CAN DESIGN, OPTIMIZE OR CONTROL A SYSTEM
   - DESIGN, OPTIMIZATION AND CONTROL MAY REQUIRE 1,000’s OF SIMULATIONS
   - SIMULATION ASSUMES ALL PARAMETERS, INPUTS, DISTURBANCES ARE GIVEN
   - CONTROL DESIGN IS THE PROCESS OF FINDING OPTIMAL INPUTS
   - DESIGN AND CONTROL IS OFTEN DONE WITH REDUCED ORDER MODELS

- CONCATENATING THE “BEST SIMULATION TOOL” WITH THE “BEST DESIGN TOOL” PRODUCES THE BEST DESIGN OR CONTROL

- IF I CAN SIMULATE THE OPEN-LOOP SYSTEM, THEN I CAN SIMULATE THE CLOSED-LOOP SYSTEM

- THERE ARE NO THEORIES TO DEAL WITH MULTI-SCALE DISTRIBUTED PARAMETER CONTROL SYSTEMS
NEW DOE HUB

August 22, 2010: The U.S. Department of Energy announced the establishment of a research hub for energy-efficient building technologies at the Navy Yard in South Philadelphia. The Energy-Efficient Building Systems Design Hub’s mission will be to develop new energy efficient components for integration into whole building systems, new mathematical models and computer design tools that can be used for design and control of new buildings and to retrofit existing buildings. It also will analyze the role of policy, markets and behavior in the adoption and use of energy technology in buildings.
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Task 1: MODELING AND COMPUTATIONAL TOOLS SPECIFICALLY FOR DESIGN, OPTIMIZATION AND CONTROL OF HIGH PERFORMANCE BUILDINGS ....

Interdisciplinary Center for Applied Mathematics
Difficult but Essential R&D

- NEW MATHEMATICAL AND COMPUTATIONAL TOOLS SPECIFICALLY FOR DESIGN – CONTROL - OPTIMIZATION – SENSITIVITY – UNCERTAINTY
  - BASED ON STATE OF THE ART COMPUTATIONAL SCIENCE
  - DEAL WITH MULTI-SCALE PHYSICS, CHEMISTRY AND COMPLEXITY
  - HAVE USER INTERFACES SUITABLE FOR BROAD BUILDING STOCKS
  - BE BUILT ON OPEN SOURCE SOFTWARE

**ROLES OF HIGH PERFORMANCE COMPUTING**

- WHOLE BUILDING MODELING, SIMULATION AND HOLISTIC DESIGN
- HPC ENABLED MODEL REDUCTION
- HPC FOR PARALLEL OPTIMIZATION & DESIGN
- PREDICTIVE MODELING AND SIMULATION OF CLOSED-LOOP DYNAMICS
  - ENABLE “PLUG-AND-PLAY” FOR NEW COMPONENT TECHNOLOGIES
  - SHOULD “SOLVE” NON-STANDARD EQUATIONS –
    - NON-LOCAL (IN SPACE) COUPLED SYSTEMS
    - PDE SYSTEMS IN HIGH SPATIAL DIMENSION (≥ 6)
    - ALLOW FOR DATA DRIVEN COMPUTATION
    - UNCERTAINTY QUANTIFICATION
Difficult but Essential R&D

NEW MATHEMATICAL AND COMPUTATIONAL TOOLS **SPECIFICALLY** FOR DESIGN – CONTROL - OPTIMIZATION – SENSITIVITY – UNCERTAINTY

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HP2C Enabled Model Reduction

High Productivity Computing

High Fidelity Physics-Based Simulation Tools
\[
\begin{align*}
\frac{\partial}{\partial t} \theta + u \cdot \nabla \theta &= \kappa \Delta \theta \\
\frac{\partial}{\partial t} u + u \cdot \nabla u + \nabla p &= \nu \Delta u + \beta \theta \varepsilon_3 \\
\nabla \cdot u &= 0 \\
u|_{t=0} &= u_0, \quad \theta|_{t=0} &= \theta_0.
\end{align*}
\]

Cluster, Grid, Cloud

ADVANCED MODEL REDUCTION METHODS

HP2C ENABLED MODELING & COMPUTER DESIGN TOOLS

INTERACTIVE

Hierarchical & Reduced-Order Design and Control Models
\[
\dot{x}_n = f_n(x_n, u, q) + v
\]

BUILDING PHYSICS DESIGN PARAMETERS

Building Information Model
- Architecture, Materials
- HVAC, Electricity, Loads
- Occupancy
- Sensors, Actuators ...

INTEROPERABLE

COMPUTER DESIGN TOOL

Chris Byrnes Workshop
HP2C REQUIREMENTS

Whole Building Simulations
(complex geometry, multiple sub-systems, and realistic indoor - external uncertainties)

Multi-zone Building Simulation
(simplified geometry and boundary conditions)

Thermal Environment in an Individual Zone/Room

1 - 10 TFlops machine*  
1 PFlops machine*  
10 PFlops machine*

* Less than 1 hour turnaround for practical design calculations
High Performance Computing for:

- Holistic control design - optimal sensor placement

**Observation**

- Functional gain
- Sensor on lower side wall

**Shape sensitivity analysis and uncertainty**

**Sensitivity with respect to wall movement**
Sensor Location Issues

\[ \rho C \left( \frac{\partial}{\partial t} T(t, \vec{x}) + \nu(t, \vec{x}) \cdot \nabla T(t, \vec{x}) \right) = \kappa \nabla^2 T(t, \vec{x}) + g(\vec{x}) \nu(t) \]

\[ T(t, \vec{x}) \bigg|_{\Gamma_1} = b(\vec{x}) u(t) \]

\[ \xi(t) = DT(t, \cdot) = \iiint_{\Omega_D} d(\vec{x}) T(t, \vec{x}) d\vec{x} \]

\( u(t) = \) Temperature of inflow

\( \Omega \) Controlled region

\( \Gamma_1 \)
Sensor Location Issues

Sensor region

Controlled region

\[ u(t) = \text{Temperature of inflow} \]

\[ \Gamma_1 \]

\[ \Omega(q) = \{ x \in \Omega : \| q - \bar{x} \| < \delta \} \]

\[ \rho C \left( \frac{\partial}{\partial t} T(t, \bar{x}) + v(t, \bar{x}) \cdot \nabla T(t, \bar{x}) \right) = \kappa \nabla^2 T(t, \bar{x}) + g(\bar{x})v(t) \]

\[ T(t, \bar{x}) \big|_{\Gamma_1} = b(\bar{x})u(t) \]

\[ \xi(t) = DT(t, \cdot) = \iiint_{\Omega_D} d(\bar{x})T(t, \bar{x})d\bar{x} \]

\[ y(t) = C(q)T(t, \cdot) = \iiint_{\Omega(q)} c(\bar{x})T(t, \bar{x})d\bar{x} + w(t) \]
Distributed Parameter Formulation

\[ \dot{z}(t) = Az(t) + Bu(t) + Gv(t) \]

\[ \xi(t) = Dz(t) \]

\[ y(t) = C(q)z(t) + Ew(t) \]

**LINEAR DP SYSTEM**

**CONTROLLED OUTPUT**

**SENSED OUTPUT**

ALL CONTINUOUS (Hilbert-Schmidt) LINEAR FEEDBACK LAWS HAVE THE REPRESENTATION

\[ u(t) = -Kz(t) = -\iiint_{\Omega} k_T(\tilde{x})T(t, \tilde{x})d\tilde{x} \]
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\[ u(t) = -Kz(t) = -\int\int\int_{\Omega} k_T(\tilde{x})T(t, \tilde{x})d\tilde{x} \]

**FEEDBACK FUNCTIONAL GAIN**
Dynamic Controller

\[ u(t) = -Kz(t) = -\iiint_{\Omega} k_T(\bar{\mathbf{x}}) T(t, \bar{\mathbf{x}}) d\mathbf{\bar{x}} \]

\[ \hat{u}(t) = -Kz_e(t) = -\iiint_{\Omega} k_T(\bar{\mathbf{x}}) T_e(t, \bar{\mathbf{x}}) d\mathbf{\bar{x}} \]

\[ \dot{z}_e(t) = A_e \dot{z}(t) + F(q)[y(t) - C(q)z_e(t)] \]

\[ [F(q)y](\bar{\mathbf{x}}) = f_T(q, \bar{\mathbf{x}})y \]
Dynamic Controller

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OBSERVER

FUNCTIONAL GAINS
Dynamic Controller

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**OBSERVER**

**FUNCTIONAL GAINS**

**WHAT DO THESE FUNCTIONAL GAINS TYPICALLY LOOK LIKE?**
**Functional Gains**

\[ u(t) = -\int\int\int_{\Omega} k_T(\bar{x})T(t, \bar{x})d\bar{x} \]

\[ [F(q)y](\bar{x}) = f_T(q, \bar{x})y \]

**PLACE SENSOR ON DESK**

**SENSOR ON WALL**

**FEEDBACK FUNCTIONAL GAIN**

**WHICH WALL AND WHERE**

**OBSERVER FUNCTIONAL GAIN SENSOR ON TOP WALL**
Where to Place Sensors

\[ u(t) = -\iiint_{\Omega} k_T(\tilde{x})T(t, \tilde{x})d\tilde{x} \]

Let \( \Omega_K \subset \Omega \) be the "support" of \( k_T(\tilde{x}) \)

\[ u(t) = -\iiint_{\Omega} k_T(\tilde{x})T(t, \tilde{x})d\tilde{x} \cong -\iiint_{\Omega_K} k_T(\tilde{x})T(t, \tilde{x})d\tilde{x} \]

NEED (SENSED) INFORMATION ABOUT \( T(t, \tilde{x}) \) ON \( \Omega_K \)

MAYBE THE REGION \( \Omega_K \subset \Omega \) IS A GOOD PLACE TO LOCATE SENSORS?
Where to Place Sensors

\[ u(t) = -\int\int\int_{\Omega} k_T(\vec{x})T(t, \vec{x}) d\vec{x} \]

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MAYBE THE REGION \( \Omega_K \subset \Omega \) IS A GOOD PLACE TO LOCATE SENSORS?

WE HAVE DONE THIS AND IT (OFTEN) WORKS, BUT

--- NOT ALWAYS POSSIBLE ---

--- NOT ALWAYS THE BEST LOCATION ---
Observer Functional GainS

Sensor on centered on lower side wall

Sensor on lower left side wall

OBSERVER FUNCTIONAL GAIN SENSOR ON SIDE WALL

OBSERVER FUNCTIONAL GAIN SENSOR ON SIDE WALL
Observer Functional GainS

Sensor on centered on lower side wall

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PROVIDES ENGINEERING INSIGHT BUT CAN THIS BE OBTAINED THROUGH FORMAL OPTIMIZATION?

OBSERVER FUNCTIONAL GAIN SENSOR ON SIDE WALL

OBSERVER FUNCTIONAL GAIN SENSOR ON SIDE WALL
State Estimator

\[ M = GG^*, \quad Q = D^*D, \quad \mathcal{O} = CC^*, \quad N = EE^* = 1 \]

\[ A\Sigma + \Sigma A^* - \Sigma C(q)^* C(q)\Sigma + GG^* = 0 \]

\[ F(q) = \Sigma C(q)^* = \Sigma(q)C(q)^* \]

**IF** \( \mathcal{O}(q) = C(q)C(q)^* \) **AND** \( M = GG^* \) **ARE NUCLEAR**

**THEN** \( \Sigma = \Sigma(q) \) **AND** \( F(q) \) **ARE NUCLEAR**

2005 - Curtain, Mikkola and Sasane, JMAA

1972 – Bensoussan, Springer Lecture Notes 294
Optimal Sensor Location Problem

\[ \dot{z}_e(t) = Az_e(t) + Bu(t) + F(q)[y(t) - C(q)z_e(t)] \]

\[ F(q) = \Sigma C(q)^* \]

\[ A\Sigma + \Sigma A^* - \Sigma C(q)^* C(q)\Sigma + GG^* = 0 \]

Here, \( \Sigma = \Sigma(q) \) is the state estimation covariance operator and the estimation error is

\[ err(q) = trace\Sigma(q) \]
Optimal Sensor Location Problem

\[
\dot{z}_e(t) = A z_e(t) + B u(t) + F(q)[y(t) - C(q)z_e(t)]
\]

\[
F(q) = \Sigma C(q)^*
\]

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A\Sigma + \Sigma A^* - \Sigma C(q)^* C(q)\Sigma + GG^* = 0
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Here, \( \Sigma = \Sigma(q) \) is the state estimation covariance operator and the estimation error is

\[
err(q) = \text{trace} \Sigma(q)
\]

\[
J(q) = \text{trace} \Sigma(q) + N(q), \quad q \in \Lambda \subseteq \hat{\Omega}
\]

Find \( q^{opt} \in \Lambda \) to minimize \( J(q) \)
\[ g(x) = 1, \quad M = GG^* \]

\[ 1 / \text{trace}(\Sigma_h(q)) \]
\[ g(x) = 1, \ M = GG^* \]

MATCHES INSIGHT OBTAINED FROM FUNCTIONAL GAINS

BASIS FOR A RIGOROUS METHODOLOGY & COMPUTATIONS

\[ \frac{1}{\text{trace}(\Sigma_h(q))} \]
The Riccati Equation

**RICCATI DIFFERENTIAL EQUATION**

\[ \dot{\Sigma}(t) = A\Sigma(t) + \Sigma(t)A^* - \Sigma(t)C(\dot{\gamma}(t))^*C(\dot{\gamma}(t))\Sigma(t) + GG^* \]

\[ \Sigma(0) = \Sigma_0 \]
The Riccati Equation

**RICCATI DIFFERENTIAL EQUATION**

\[
\dot{\Sigma}(t) = A\Sigma(t) + \Sigma(t)A^* - \Sigma(t)C(\vec{y}(t))^* C(\vec{y}(t))\Sigma(t) + GG^*
\]

\[
\Sigma(0) = \Sigma_0
\]

**RICCATI INTEGRAL EQUATION**

\[
\Sigma(t) = S(t)\Sigma_0 S^*(t)
\]

\[
+ \int_0^t S(t-s)G(s)G^*(s) - \Sigma(s)C(\vec{y}(s))^* C(\vec{y}(s))\Sigma(s))S^*(t-s)\,d
\]
The Riccati Equation

**RICCATI DIFFERENTIAL EQUATION**

\[ \dot{\Sigma}(t) = A\Sigma(t) + \Sigma(t)A^* - \Sigma(t)C(\vec{y}(t))^*C(\vec{y}(t))\Sigma(t) + GG^* \]

\[ \Sigma(0) = \Sigma_0 \]

**RICCATI INTEGRAL EQUATION**

\[ \Sigma(t) = S(t)\Sigma_0S^*(t) \]

\[ + \int_0^t S(t-s)(G(s)G^*(s) - \Sigma(s)C(\vec{y}(s))^*C(\vec{y}(s))\Sigma(s))S^*(t-s)ds \]

**HOW DOES ONE INTERPRET THE INTEGRAL?**

**USUALLY POINTWISE**

**BOCHNER INTEGRAL IS IMPORTANT FOR NUMERICS**
Riccati Integral Equation

Theorem (Rautenberg) Let $H$ be a separable Hilbert space and assume

(i) $S(t)$ is a $C_0$-semigroup

(ii) $\Sigma_0 \in \mathcal{J}_1$ (The space of Nuclear Class Operators)

(iii) $\mathcal{O}(\cdot) = \mathcal{O}^*(\cdot) \in L^1_{loc}([0, +\infty); \mathcal{J}_1)$

(iv) $\mathcal{M}(\cdot) = \mathcal{M}^*(\cdot) \in L^\infty_{loc}([0, +\infty); \mathcal{J}_1)$.

Then the mapping $F : L^1_{loc}([0, +\infty); \mathcal{J}_2) \longrightarrow C([0, +\infty); \mathcal{J}_1)$

$$F(\Sigma(\cdot)) = S(t)\Sigma_0 S^*(t)$$

$$+ \int_0^t S(t-s)(\mathcal{O}(s) - \Sigma(s) \mathcal{M}(s)\Sigma(s)) S^*(t-s) \; ds$$

is well defined, where the integral is the Bochner integral.
Theorem (Rautenberg) Let $H$ be a separable Hilbert space and assume

(i) $S(t)$ is a $C_0$-semigroup

(ii) $\Sigma_0 \in \mathcal{L}$ (The space of Nuclear Class Operators)

Then the mapping $F : L^1_{loc} ([0, +\infty); \mathcal{J}_2) \longrightarrow C([0, +\infty); \mathcal{J}_1)$

$$F(\Sigma(\cdot)) = S(t)\Sigma_0 S^*(t)$$

$$+ \int_{0}^{t} S(t-s)(\mathcal{O}(s) - \Sigma(s) M(s) \Sigma(s)) S^*(t-s) d$$

is well defined, where the integral is the Bochner integral.
A 3D Example

\[ \tilde{\kappa}(t, x, y, a) = \begin{bmatrix} 20 & 20 & 20 \end{bmatrix}^T \quad g(x, y, z) = 100 \]

The Bochner integrals that define \( GG^* \) and \( C \) are computed with tolerances of \( 10^{-7} \) and \( 10^{-3} \), respectively.
(c) Isosurface for $J(x, y, z) \approx 61$

(d) Isosurface for $J(x, y, z) \approx 74$
(c) Isosurface for $J(x, y, z) \approx 61$

(d) Isosurface for $J(x, y, z) \approx 74$

(e) Isosurface for $J(x, y, z) \approx 87$

(f) Isosurface for $J(x, y, z) \approx 118$
A 3D Example

\[ \tilde{\kappa}(t, x, y, a) = \begin{bmatrix} 20 & 20 & 20 \end{bmatrix}^T \quad g(x, y, z) = 2 \]

The Bochner integrals that define $GG^*$ and $C$ are computed with tolerances of $10^{-6}$ and $10^{-2}$, respectively.
(c) Isosurface for $J(x, y, z) \simeq 2.34$

(d) Isosurface for $J(x, y, z) \simeq 2.4$
(c) Isosurface for $J(x, y, z) \simeq 2.34$

(d) Isosurface for $J(x, y, z) \simeq 2.4$

(e) Isosurface for $J(x, y, z) \simeq 2.5$

(f) Isosurface for $J(x, y, z) \simeq 2.53$
ACCURATE APPROXIMATION OF THE BOCHNER INTEGRALS IS ESSENTIAL

(c) Isosurface for $J(x, y, z) \simeq 2.34$
(d) Isosurface for $J(x, y, z) \simeq 2.4$

APPLIED & COMPUTATIONAL MATHEMATICS ARE THE ENABLING SCIENCES

(e) Isosurface for $J(x, y, z) \simeq 2.5$
(f) Isosurface for $J(x, y, z) \simeq 2.53$
Wisdom of Burns and Byrnes

“If stupidity got us into this mess, then why can’t it get us out”

-- Will Rogers

“We can’t solve (today’s) problems by using the same kind of thinking we used when we created them.”

-- Albert Einstein
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-- Albert Einstein

“A man only learns in two ways, one by reading, and the other by association with smarter people”

-- Will Rogers
John Burns’ Confession

I don’t read much, but I have always associated with much smarter people...
... and none were smarter than Chris.
John Burns’ Confession

I don’t read much, but I have always associated with much smarter people
... and none were smarter than Chris.

THANKS CHRIS
WE MISS YOU