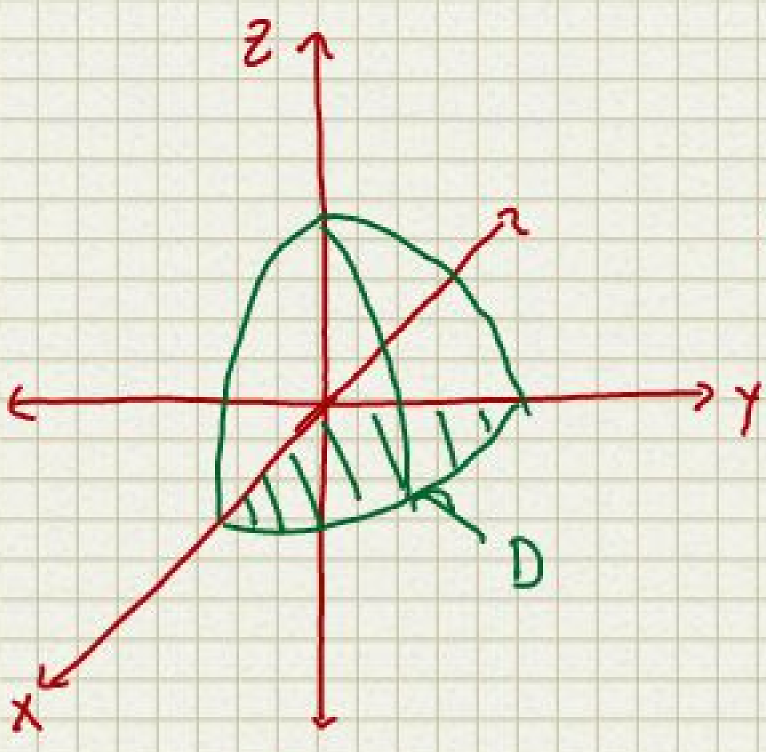
5/1/2014 Final Review Cont.

Find SA of $z = 8 - 2x^2 - 2y^2$ $\quad x \geq 0$, $y \geq 0$, $z \geq 0$



$$SA = \iint_S dS = \iint_D \sqrt{1 + z_x^2 + z_y^2}\, dA$$

$z = 0$

$z = 8 - 2x^2 - 2y^2 = 0$

$2(x^2 + y^2) = 8 \quad x^2 + y^2 = 4$

$z_x = -4x \quad z_y = -4y$

$$SA = \iint_D \sqrt{1 + 16x^2 + 16y^2}\, dA$$

$$= \int_0^{\pi/2} \int_0^2 r\sqrt{1 + 16r^2}\, dr\, d\theta = \frac{1}{32} \int_0^{\pi/2} \int_{-}^{-} u^{1/2}\, du\, d\theta$$

$u = 16r^2 + 1$

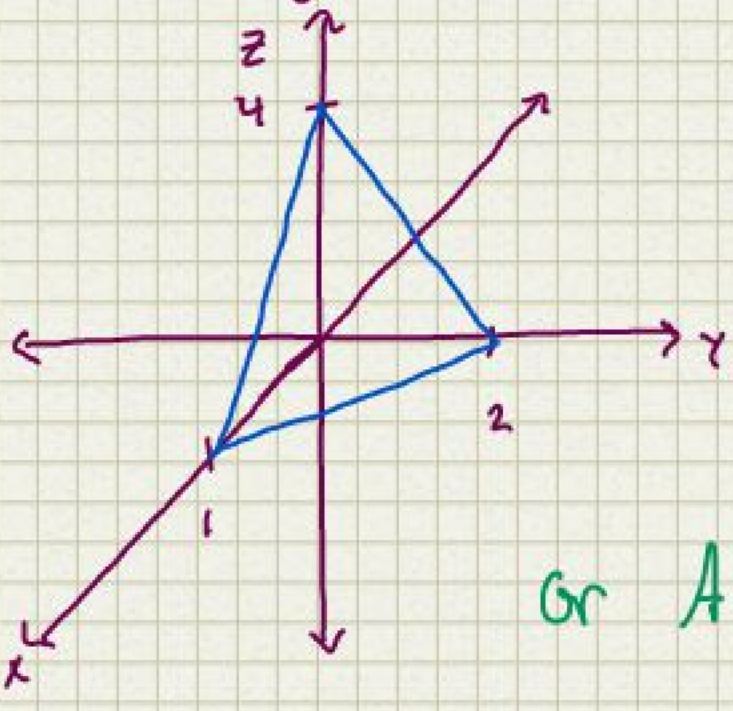
$du = 32r\, dr$

$$= \frac{1}{32} \int_0^{\pi/2} \frac{2}{3} u^{3/2} \Big|_{-}^{-} d\theta = \frac{1}{48}\, \theta \Big|_0^{\pi/2} (16r^2 + 1)^{3/2} \Big|_0^2$$

$$= \frac{1}{48} \cdot \frac{\pi}{2} \left[(16(4) + 1)^{3/2} - (1)^{3/2} \right]$$

Tetrahedron

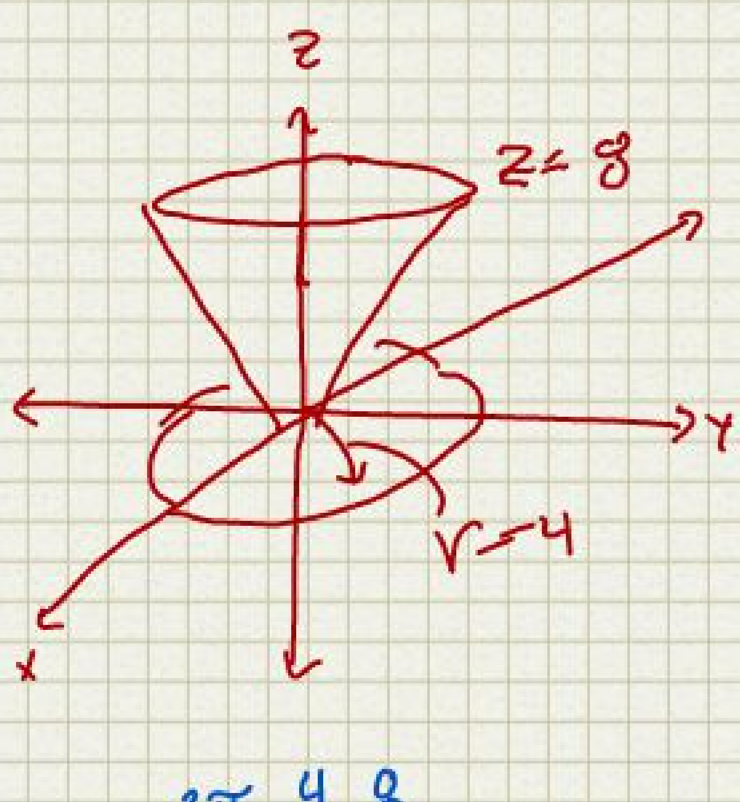
$4x + 2y + z = 4 \qquad x=0,\ y=0,\ z=0$



$\forall = \iiint_{\forall} d\forall$

$= \int_0^1 \int_{y=0}^{y=2-2x} \int_0^{z=4-4x-2y} dV$

Or $A = \frac{1 \cdot 2}{2} \quad \forall = \frac{A \cdot h}{3} = \frac{1}{3}(1 \cdot 4) = \frac{4}{3}$



$I = \iiint_D (3x^2 + 3y^2)\, d\forall$

$z \begin{cases} \text{inside the cone } z = \sqrt{4x^2+4y^2} \\ \text{below } z = 8 \end{cases}$

Cylindrical Coordinates

$z = \sqrt{4r^2} = 2r$

$z = 8$

$I = \int_0^{2\pi} \int_0^4 \int_{2r}^{8} 3r^2\, r\, dz\, dr\, d\theta = \int_0^{2\pi} \int_0^4 3r^3(8-2r)\, dr\, d\theta$

$= 6\pi \left(\frac{8}{4} r^4 - \frac{2}{5} r^5 \right) \Big|_0^4 = 6\pi \left(2(4^4) - \frac{2}{5}(4^5) \right)$

$\vec{F} = e^x y\,\hat{\imath} + z\cos(y)\,\hat{\jmath} + xy\,\hat{k}$

$\text{div}\,\vec{F} = \nabla \cdot F = e^x y + z(-\sin(y)) + 0$

$$\text{curl}\,\vec{F} = \nabla \times F = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial x & \partial y & \partial z \\ e^x y & z\cos(y) & xy \end{vmatrix}$$

$$= \hat{\imath}(x - \cos(y)) - \hat{\jmath}(y) + \hat{k}(0 - e^x)$$

$\vec{F} = \langle e^x y^2 + x^2,\ 2e^x y + 3y \rangle$

a) $\vec{F}$ is conservative?

b) $\vec{F}$ scalar potential

c) $\oint \vec{F} \cdot d\vec{r} = 0$

A) $(2e^x y + 3y)\,dx \quad 2e^x y$

$(e^x y^2 + x^2)\,dy \quad 2e^x y \quad$ conservative

B) $\int e^x y^2 + x^2\,dx = e^x y^2 + \frac{1}{3}x^3$

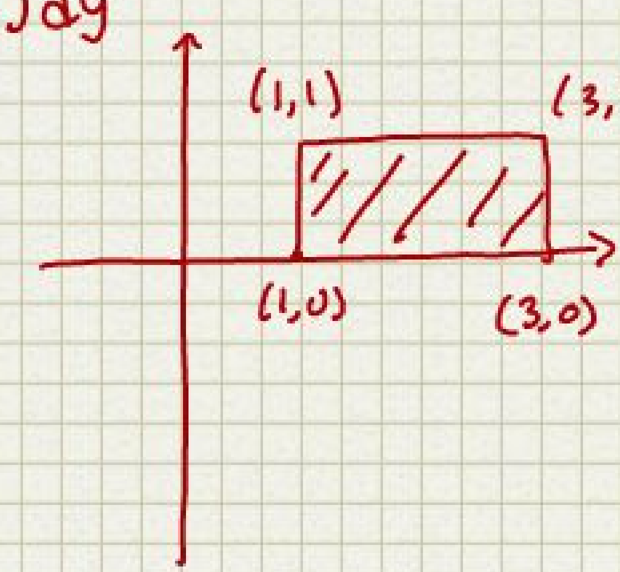
$\int 2e^x y + 3y\,dy = e^x y^2 + \frac{3}{2}y^2$

$f = e^x y^2 + \frac{1}{3}x^3 + \frac{3}{2}y^2$

$\oint_C (x\cos(x) - y)\,dx + (x + y^2\tan^{-1}(y))\,dy$

$= \iint_D (\partial_x f_2 - \partial_y f_1)\,dA$

$I = \iint_D 1 + 1\,dA = 2\iint_D dA = 4$



Use Stokes Theorm to solve Flux Integral

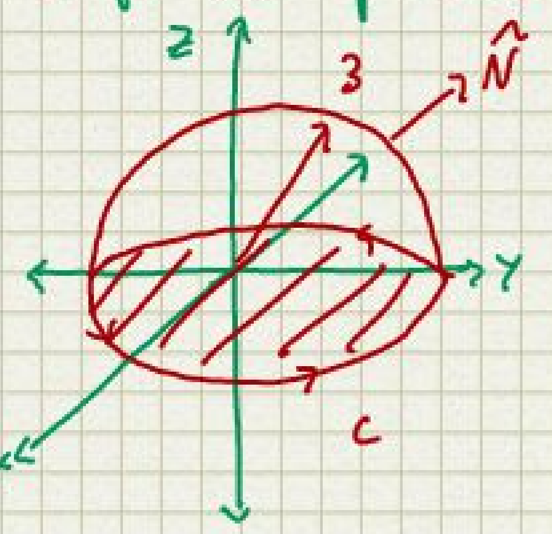
$\iint_S (\nabla \times \vec{F} \cdot \hat{N})\,dS \qquad \vec{F} = (z^2+3y)\hat{\imath} + (\cos(z)+3x)\hat{\jmath} + (xyz^2)\hat{k}$

$S: x^2+y^2+z^2 = 9 \quad r=3$

$z \geq 0 \quad \hat{N}$ points upwards

$\nabla \times \vec{F} =$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial x & \partial y & \partial z \\ z^2+3y & \cos(z)+3x & xyz^2 \end{vmatrix}$$



$= (--)\hat{\imath} - (--)\hat{\jmath} + (3-3)\hat{k}$

$= \langle --, --, 0 \rangle$

Section 9.6 (Planes)

Find vector normal to surface + tang. plane at Point

$x^2+y^2+z^2=14 \quad (3,1,2)$

$z = f(x,y)$

$\vec{N} = \langle -f_x, -f_y, 1\rangle$ (explicit)

$* \; \vec{N} = \nabla G|_p = \langle 2x, 2y, 2z\rangle|_{(3,1,2)}$ $\qquad G = z - f(x,y) = 0$

$= \langle 6, 2, 4\rangle = \vec{N} = \langle A, B, C\rangle$

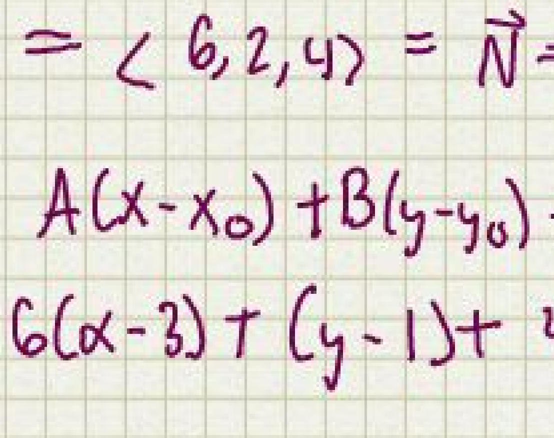
$\vec{N} = \langle \partial_x G, \partial_y G, \partial_z G\rangle$

$= \langle -f_x, -f_y, 1\rangle$

$A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$

$6(x-3) + (y-1) + 4(z-2) = 0$

$r = 2+2\cos(\theta)$



$A = \iint_D dA = \int_0^{2\pi}\int_0^{2+2\cos(\theta)} r\,dr\,d\theta$ $\qquad = 3\int_0^{2\pi} = 6\pi$

$= \int_0^{2\pi} \frac{r^2}{2}\Big|_0^{2+2\cos(\theta)} = \frac{1}{2}\int_0^{2\pi} (2+2\cos(\theta))^2\,d\theta$

$= \frac{1}{2}\int_0^{2\pi} 4 + 8\cos(\theta) + 4\cos^2(\theta)\,d\theta$

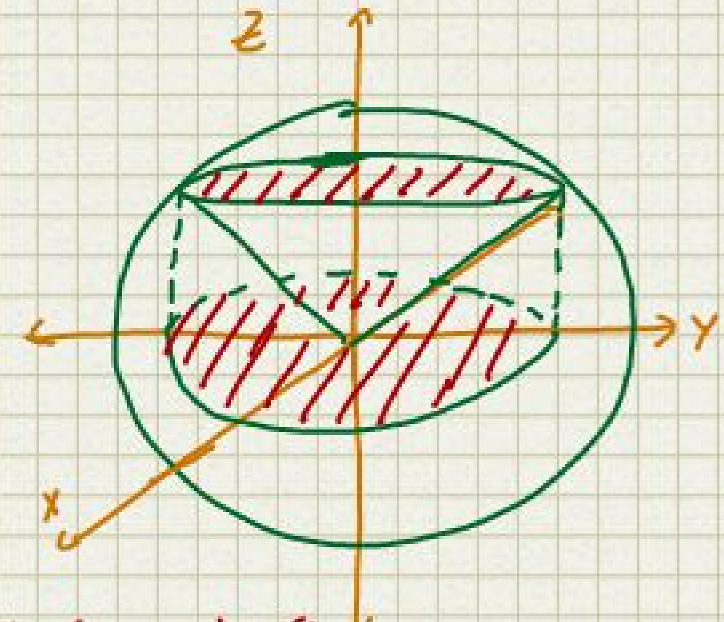
$= \int_0^{2\pi} 2 + 4\cos(\theta) + 2\cos^2(\theta)\,d\theta = \int_0^{2\pi} 2 + 4\cancel{\cos(\theta)}^{0} + \cancel{2}\cancel{(\tfrac{1}{2}}^{1}(1 - \cancel{\cos(2\theta)}^{0})$

SA? $x^2+y^2+z^2=8$ inside $z=\sqrt{x^2+y^2}$

$$SA=\iint_S dS=\iint_D \sqrt{1+z_x^2+z_y^2}\,dA$$

$$=\iint_D \frac{r}{z}dA=\iint_D \frac{2\sqrt{2}}{\sqrt{8-x^2+y^2}}\,dA$$

$$=\int_0^{2\pi}\int_0^2 \frac{2\sqrt{2}\,r}{\sqrt{8-r^2}}\,dr\,d\theta$$



Cone in sphere

$u=8-r^2$

$du=-2r\,dr$

$$=\frac{2\sqrt{2}}{(-2)}\int_0^{2\pi}\frac{1}{\sqrt{u}}\,du\,d\theta=-\sqrt{2}\int_0^{2\pi}2(8-r^2)^{1/2}\Big|_0^2 d\theta=$$

$$=-2\sqrt{2}(2\pi)\left[\sqrt{4}-\sqrt{8}\right]=-4\sqrt{2}\pi(2-2\sqrt{2})$$

Sphere w/ spherical Cord.

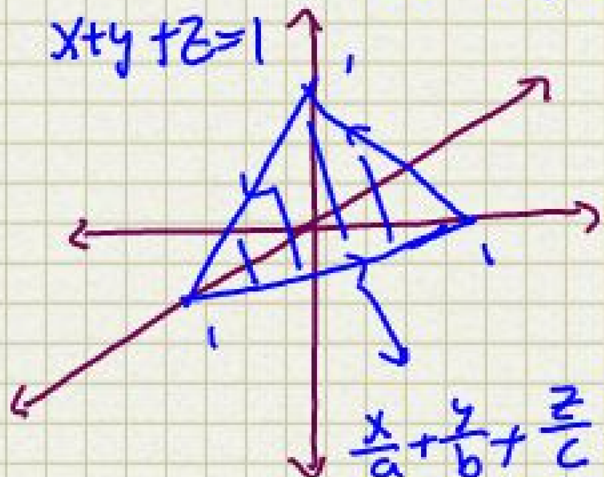
$$I=\iiint_D (x^2+y^2+z^2)\,dV=\int_0^{\pi/2}\int_0^{\pi/2}\int_0^2 \rho^2\rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta$$

$x^2+y^2+z^2\le 4$

$$=\int_0^{\pi/2}d\theta\int_0^{\pi/2}\sin(\phi)\,d\phi\int_0^2\rho^4\,d\rho=\left(\frac{\pi}{2}\right)\left(\frac{2^5}{5}\right)$$

<u>Stokes Theorm</u>

$\vec{N}=\langle 1,1,1\rangle$



$$\vec{F}=(e^{x^2}+3y)\hat{i}+(\cos(y)+x)\hat{j}+z^2\hat{k}$$

$$\oint_C \vec{F}\cdot d\vec{r}=\iint_S(\nabla\times\vec{F}\cdot\hat{N})\,dS$$

$$\nabla\times\vec{F}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \partial x & \partial y & \partial z\\ e^{x^2}+3y & \cos(y)+x & z^2\end{vmatrix}$$

$$=(0-0)\hat{i}-(0-0)\hat{j}+(1-3)\hat{k}=\langle 0,0,-2\rangle$$

$$\iint_S(\nabla\times\vec{F}\cdot\vec{N})\,dS=\iint_D\langle 0,0,-2\rangle\cdot\langle 1,1,1\rangle\,dA=-2\iint_D dA$$

$$=-2\left(\frac{1}{2}\right)=-1$$

$F(x,y) = \cos(xy)$

$x = u^2 + v^2$ $\qquad \dfrac{\partial F}{\partial u} = \dfrac{\partial F}{\partial x} x_u + \dfrac{\partial F}{\partial y} y_u$

$y = u^2 - v^2$

$\dfrac{\partial F}{\partial u} \quad \dfrac{\partial F}{\partial v}$ $\qquad \dfrac{\partial F}{\partial v} = \dfrac{\partial F}{\partial x} x_v + \dfrac{\partial F}{\partial y} y_v$

$F_x = -y\sin(xy) \qquad x_u = 2u \qquad x_v = 2v$

$F_y = -x\sin(xy) \qquad y_u = 2u \qquad y_v = -2v$

$F_x = -y\sin(xy)2u - x\sin(xy)2u \qquad F_y = -y\sin(xy)2v - x\sin(xy)(-2v)$

Use Flux $\quad \vec{F} = \langle z\sin(y), e^x + y, z + \cos(xy) \rangle$

$\vec{N}$ unit outward normal

$S: x^2 + y^2 + z^2 = 1$

$$\iint_S \vec{F} \cdot \vec{N}\, dS = \iiint_V (0+1+1)\, dV = 2\iiint_V dV = 2\left(\frac{4}{3}\pi\right)$$

$$= \frac{8}{3}\pi$$