4/30/2014

Final Monday May 12 4:30 → 7:00, Chem 113
Orange scantron, blue book, ID, pencil, No calc
Formula sheet.
12 MP 4 essay (show work)

Review

$\vec{F} = 3t^2\hat{\imath} - \sin(t)\hat{\jmath} + e^t\hat{k}$
$\vec{G} = \cos(t)\hat{\imath} + t^{-2}\hat{k}$

$\vec{F}\cdot\vec{G} = 3t^2\cos(t) + e^t t^{-2}$

$$\vec{F}\times\vec{G} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3t^2 & -\sin(t) & e^t \\ \cos(t) & 0 & t^{-2} \end{vmatrix} = \left(\hat{\imath}\left(-\sin(t)\cdot\frac{1}{t^2}\right) - \hat{\jmath}\left(3 - e^t\cos(t)\right) + \hat{k}\ \sin(t)\cos(t)\right)$$

$\vec{v} = e^{2t}\hat{\imath} - \sin(t)\hat{\jmath} + 2t\hat{k}$ $\quad \vec{r}(t)$ $\quad \vec{r}(0) = \langle 1, 2, 1\rangle$

$\vec{a}(t) = \vec{v}\,'(t) = \langle 2e^t, -\cos(t), 2\rangle$

$\vec{r}(t) = \int \vec{v}(t)\,dt + \vec{c} = \langle \frac{e^{2t}}{2}, \cos(t), \frac{t^2}{t}\rangle$

$\vec{r}(0) = \langle 1, 2, -1\rangle = \langle \frac{e^0}{2}, \cos(0), \frac{0}{2}\rangle + \vec{c} = \langle \frac{1}{2}, 1, 0\rangle + \vec{c}$

$\vec{c} = \langle 1, 2, 1\rangle - \langle 1/2, 1, 0\rangle = \langle \frac{1}{2}, 1, -1\rangle$

$\vec{R}(t) = e^t\cos(t)\,\hat{i} + e^t\sin(t)\,\hat{j}$

- $\vec{T}$ unit tangent vector $\dfrac{\vec{R}'}{\|\vec{R}'\|}$

- $\vec{N}$ p.u.n.v $= \dfrac{\vec{T}'}{\|\vec{T}'\|}$

- K curvature $\rightarrow \left\|\dfrac{d\vec{T}(s)}{ds}\right\|$ $\rightarrow \dfrac{\|\vec{R}' + \vec{R}''\|}{\|\vec{R}'\|^3}$ $\rightarrow \dfrac{\|\vec{T}'(t)\|}{\|\vec{R}'(t)\|}$

$\vec{R}'(t) = \langle e^t\cos(t) + e^t(-\sin(t)),\ e^t\sin(t) + e^t\cos(t)\rangle$

$= e^t\langle \cos(t) - \sin(t), \sin(t) + \cos(t)\rangle$

$\|\vec{R}'\| = e^t\sqrt{(\cos(t) - \sin(t))^2 + (\cos(t) + \sin(t))^2}$

$= e^t\sqrt{2}$

$\vec{T} = \dfrac{1}{\sqrt{2}}\langle \cos(t) - \sin(t), \sin(t) + \cos(t)\rangle$

$\vec{T}' = \dfrac{1}{\sqrt{2}}\langle -\sin(t) - \cos(t), \cos(t) - \sin(t)\rangle$

$\|\vec{T}'\| = \dfrac{1}{\sqrt{2}}(\sqrt{2}) = 1$

$\vec{N} = \dfrac{\vec{T}'}{\|\vec{T}'\|} = \dfrac{\frac{1}{\sqrt{2}}}{1} = \dfrac{1}{\sqrt{2}}$

$$\frac{\partial Z(x,y)}{\partial x} \qquad F(x,y,z)=0$$

$\partial x\,(xy^2 + yz^2 + zx^2) = 0 \longrightarrow \quad y^2 + y\,2z\,z_x + z_x x^2 + z\,2x = 0$

$z_x(2yz + x^2) = -y^2 - 2xz$

$$z_x = \frac{-y^2-2xz}{2yz+x^2}$$

$2xy + z^2 + y\,2z\,z_y + z_y x^2 = 0$

$$z_y = \frac{-2xy - z^2}{2yz+x^2}$$

$$z_x = -\frac{F_x}{F_z} \qquad z_y = -\frac{F_y}{F_z}$$

$$= -\frac{y^2+2xz}{2yz+x^2}$$

$f(x,y) = e^{2x+3y} \quad P(1,0)$

$\vec{v} = 3i - 4j$

a) $D_{\vec{u}} f(p)$ b) m R change f at $p = \|\nabla f(p)\| = \sqrt{13}\,e^2$

$\|\vec{u}\|=1 \qquad \vec{u} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{1}{5}\langle 3,-4\rangle$

$D_{\vec{u}} f(x,y) = \nabla f \cdot \vec{u} = \langle 2e^{2x+3y}, 3e^{2x+3y}\rangle \cdot \frac{1}{5}\langle 3,-4\rangle$

$= -\frac{6}{5}e^2$

$f(x,y) = 2x^3 - 6xy + 3y^2 + 5$

Find and classify critical points

$\begin{cases} f_x = 0 \\ f_y = 0 \end{cases}$ $\quad \Delta = f_{xx} f_{yy} - f_{xy}^2$

$\Delta(P) = \begin{cases} >0 & \begin{cases} R\min & f_{xx} > 0 \\ R\max & f_{xx} < 0 \end{cases} \\ 0 & \text{inconclusive} \\ <0 & SP. \end{cases}$

$\begin{cases} f_x = 6x^2 - 6y = 0 \\ f_y = -6x + 6y = 0 \end{cases}$

$\begin{cases} x = y \\ x^2 - x = 0 \\ x(x-1) = 0 \end{cases}$ $\quad \begin{cases} y = 0 \\ x = 0 \end{cases}$ $\quad P_1 \begin{cases} y = 1 \\ x = 1 \end{cases}$

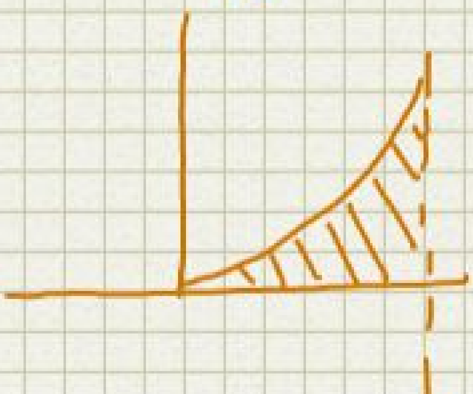
$f_{xx} = 12x \quad f_{xy} = -6 \quad f_{yy} = 6 \quad \Delta = 12x(6) - (-6)^2$

$= 6^2(2x-1)$

$\Delta(0,0) = 6^2(-1) < 0$ SP

$\left.\begin{array}{l} \Delta(1,1) = 6^2(2-1) > 0 \\ f_{yy}(1,1) = 6 > 0 \end{array}\right\} R_{\min}$

$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{1+x^3}\, dx\, dy \qquad x = \sqrt{y} \quad y = x^2$ (Reversing Integration)



$\int_0^1 \int_{y=0}^{y=x^2} \sqrt{1+x^3}\, dy\, dx = \int_0^1 \sqrt{1+x^3}\, x^2 dx$

$u = 1 + x^2$

$du = 2x^2$

$= \int \sqrt{u} \frac{du}{3} = \frac{1}{3} \frac{2}{3} (1+x^3)^{3/2} \Big|_0$

$= \frac{2}{9}(1+x^3)^{3/2}$