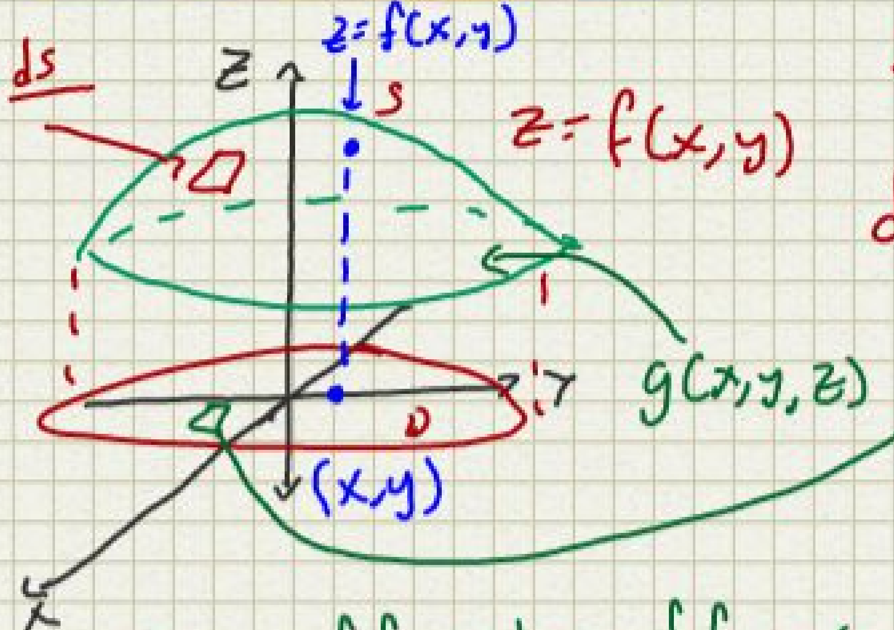
4/16/2014

13.5 Flux Integrals



$$SA = \iint_S dS = \iint_D \sqrt{1 + z_x^2 + z_y^2}\, dA$$

$$dS = J\, dA = \sqrt{1 + z_x^2 + z_y^2}\, dA$$

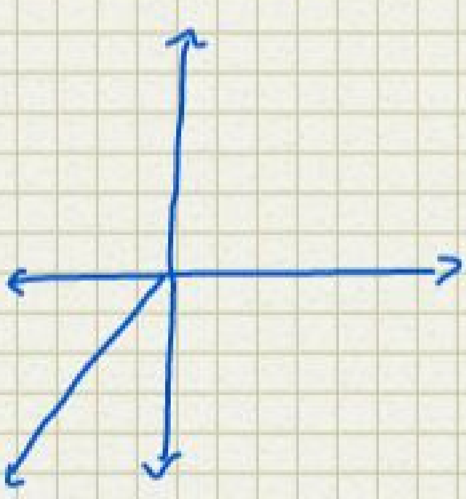
$$SI = \iint_S g\, dS = \iint_D g(x,y, f(x,y)) \sqrt{1 + f_x^2 + f_y^2}\, dA$$

Ex

$\iint_S g\, dS$

$g(x,y,z) = xz + zx^2 - 3xy$

$2x - 3y + z = 6$, $R:\ 2 \le x \le 3$
$\qquad\qquad\qquad\quad 2 \le y \le 3$



$z = f(x,y) = 6 - 2x + 3y$

$f_x = -2 \quad f_y = 3$

$\sqrt{1 + f_x^2 + f_y^2} = \sqrt{1+4+9} = \sqrt{14}$

$$\iint_S g\, dS = \iint_R (x(6-2x+3y) + 2x^2 - 3xy)\sqrt{14}\, dA$$

$$= \iint_R (6x - \cancel{2x^2} + \cancel{3xy} + \cancel{2x^2} - \cancel{3xy})\sqrt{14}\, dA$$

$$\sqrt{14}\int_2^3\int_2^3 6x\, dx\, dy = \sqrt{14}\int_2^3 6x\, dx \int_2^3 dy = \frac{6}{2}x^2\Big|_2^3$$

$$= 3\sqrt{14}(9-4) = 15\sqrt{14}$$

$$\iint_S z\,ds$$

$S = (x^2+y^2+z^2) = 4 \quad z \geq 0$

$z = \sqrt{4-x^2-y^2}$

$\sqrt{1+z_x^2+z_y^2}$ Implicit Diff

$2x + 2z\,z_x = 0$

$z_x = \dfrac{-x}{z} \quad z_y = \dfrac{-y}{z}$

$$\sqrt{1+z_x^2+z_y^2} = \sqrt{1+\frac{x^2}{z^2}+\frac{y^2}{z^2}} = \sqrt{\frac{z^2+x^2+y^2}{z^2}} = \sqrt{\frac{4}{z^2}} = \frac{2}{z}$$

$$\iint_S z\,ds = \iint_D z\left(\frac{2}{z}\right)dA = 2\iint_D dA = 2\pi(2)^2 = 8\pi$$

Hard way

$z = \sqrt{4-x^2-y^2} \quad z_x = \dfrac{1}{2}\dfrac{-2x}{\sqrt{4-x^2-y^2}} = \dfrac{-x}{\sqrt{4-x^2-y^2}}$

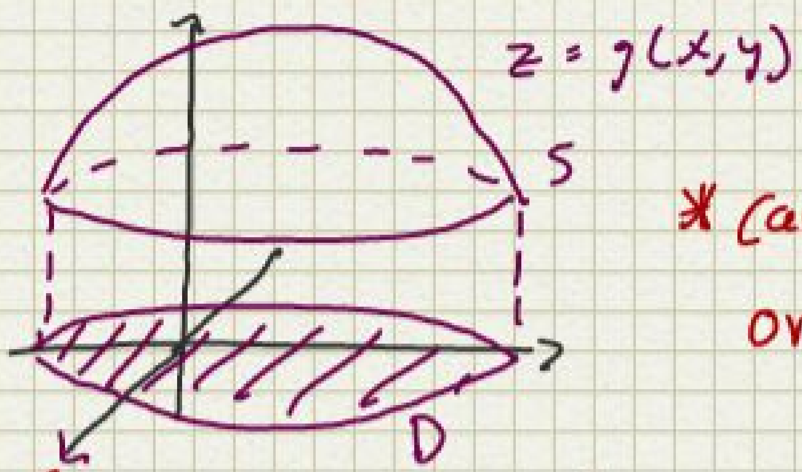
$z_y = \dfrac{-y}{\sqrt{4-x^2-y^2}}$

$$\sqrt{1+\frac{x^2}{4-x^2-y^2}+\frac{y^2}{4-x^2-y^2}} = \sqrt{\frac{4-x^2-y^2+x^2+y^2}{4-x^2-y^2}}$$

$$= \sqrt{\frac{4}{4-x^2-y^2}} = \frac{2}{\sqrt{4-x^2-y^2}}$$

Flux Integral

$\vec{F} = \langle f_1(x,y,z), f_2(x,y,z), f_3(x,y,z) \rangle$



* can't use a flux integral on

$\iint_S \vec{F} \cdot d\vec{S} = \iint_S \underbrace{\vec{F} \cdot \vec{N}}_{\text{Flux}} \, \underbrace{\vec{N}}_{\text{The direction}} \underbrace{dS}$ upward normal

$G = z - g(x,y) = 0$

$FI = \iint_D \vec{F}(x,y,g(x,y)) \cdot \dfrac{\langle -g_x, -g_y, 1 \rangle}{\sqrt{1+g_x^2+g_y^2}} \cdot \sqrt{1+g_x^2+g_y^2}$

$\dfrac{\nabla G}{\|\nabla G\|} = \dfrac{\langle -g_x, -g_y, 1 \rangle}{\sqrt{1+g_x^2+g_y^2}}$ upward

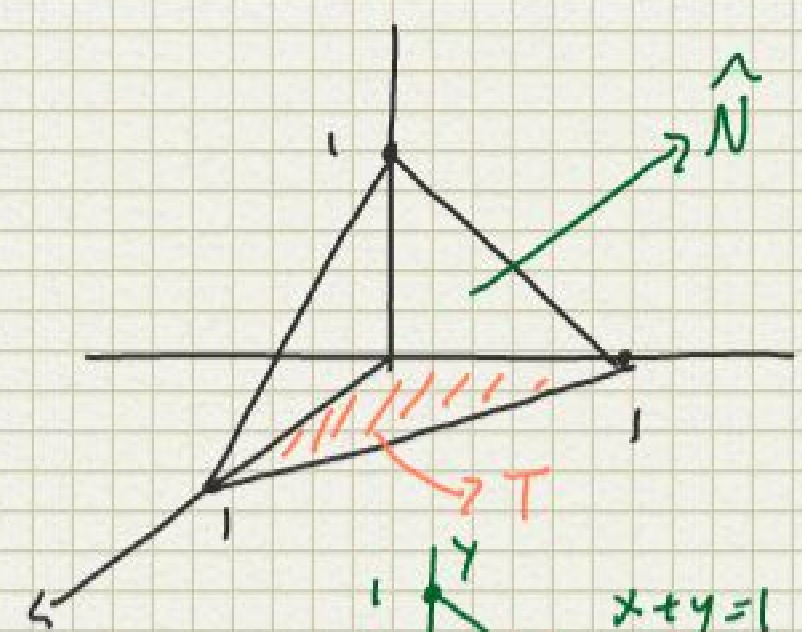
$= \iint_D \vec{F}(x,y,g(x,y)) \langle -g_x, -g_y, 1 \rangle \, dA$

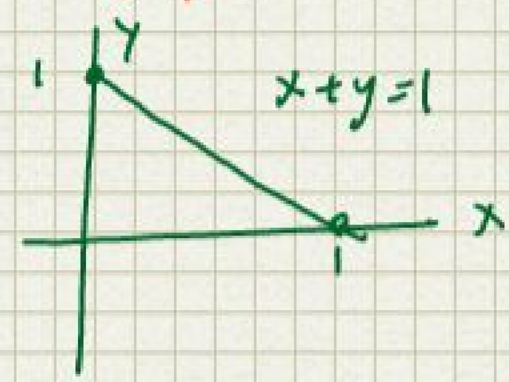
Evaluate

$$\iint_S \vec{F}\cdot\vec{N}\,ds \quad \searrow \text{upward}$$

$\vec{F} = \langle xy, z, x+y \rangle$

$S = x+y+z=1$





$z = g(x,y) = 1-x-y$

$x, y, z \geq 0 \qquad \nabla G = \langle 1,1,1 \rangle$

$$F.I. = \iint_T \langle xy, 1-x-y, x+y \rangle \langle 1,1,1 \rangle \, dA$$

$$= \iint_T (1+xy)\,dA = \int_0^1 \int_0^{1-x} xy \, dy\, dx + \frac{1}{2}$$

$$= \frac{1}{2} + \int_0^1 x \; \frac{y^2}{2}\Big|_0^{1-x} dx = \frac{1}{2} \int_0^1 \frac{x}{2}(1-2x+x^2)\,dx$$

$$= \frac{1}{2} + \frac{1}{2}\left(\frac{x^2}{2} - \frac{2x^3}{3} + \frac{x^4}{4}\right)\Big|_0^1 = \frac{1}{2} + \frac{1}{2}\left(\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right) = \ldots$$

Surface I

Flux I

$$\iint_S g(x,y,z)\,dS = \iint_D g(x,y)\, fx$$

$z = f(x,y)$

$$\iint_S \vec{F}\cdot d\vec{S} = \iint_S \vec{F}\cdot \hat{N}\,dS$$

$$= \iint_D \vec{F}(x,y,f(x,y))\cdot \frac{\langle -f_x, -f_y, 1\rangle}{\cancel{\sqrt{1+f_x^2+f_y^2}}}\cancel{\sqrt{1+f_x^2+f_y^2}}$$

$T(x,y,z) = 2x+y-3z^2$

$$\iint_S \overbrace{-k\nabla T}^{\vec{q}}\cdot d\vec{S} = -k\iint_S \langle 2,1,-6z\rangle\cdot\vec{N}\,dS$$

$S: z = 9-x^2-y^2 \quad z \geq 0$

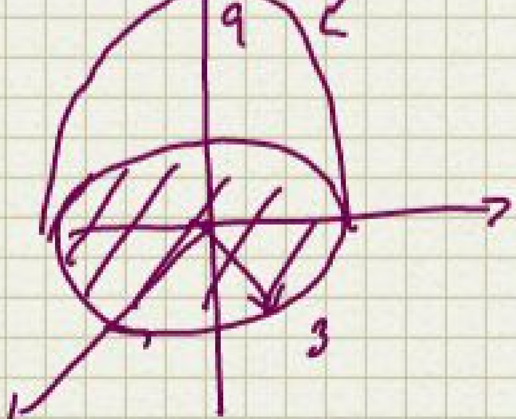
$\vec{q} = -k\nabla T = -k\langle T_x, T_y, T_z\rangle$

$= -k\langle 2,1,-6z\rangle$

upward

$z = f(x,y) = 9-x^2-y^2$

$G = z-9+x^2+y^2 \quad \nabla G = \langle 2x, 2y, 1\rangle$



$$\begin{cases} z=0 \\ z = 9-x^2-y^2 \end{cases} \Rightarrow x^2+y^2=9$$

$$FI = -k\iint_D \langle 2,1,-6(9-x^2-y^2)\rangle\langle 2x,2y,1\rangle\,dA$$

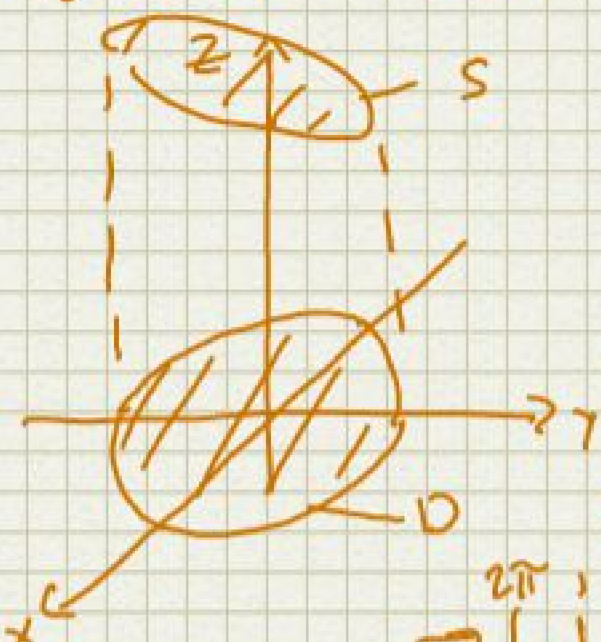
$$= -k\iint_D 4x+2y+6(x^2+y^2)-54\,dA$$

$$= -k\int_0^{2\pi}\int_0^3 4R\cos(\theta)+2R\sin(\theta)+6R^2-54\, r\,dr\,d\theta$$

$$= -k\left(-54\pi 8^2 + 6\int_0^{2\pi} d\theta \int_0^3 R^3\,dr + \left(4\int_0^{2\pi} \cos(\theta) + 2\int_0^{2\pi} \sin(\theta)\right)\int_0^3 r\,dr\right)$$

Ex

$\iint_D (x^2+y^2+z^2)\,ds$ S is the plane. $z = x+1 = f(x,y)$
inside $x^2+y^2=1$



$$\iint_S g(x,y,z)\,ds = \iint_D g(x,y,f(x,y))\sqrt{1+f_x^2+f_y^2}\,dA$$

$$= \iint_D (x^2+y^2+(x+1)^2)\sqrt{1+1^2+0^2}\,dA$$

$$= \sqrt{2}\iint_D (2x^2+2x+y^2+1)\,dA$$

$$= \sqrt{2}\int_0^{2\pi}\int_0^1 (R^2\cos^2(\theta) + 2R\underbrace{\cancel{\cos(\theta)}}_{0} + R^2 + 1)\,r\,dr\,d\theta$$

$$= -k\langle 2, 1, -6z\rangle = \sqrt{2}\left(\pi(1)^2 + \left(\int_0^1 R^3\,dR\int_0^{2\pi}(1+\cos^2(\theta))\,d\theta\right)\right)$$

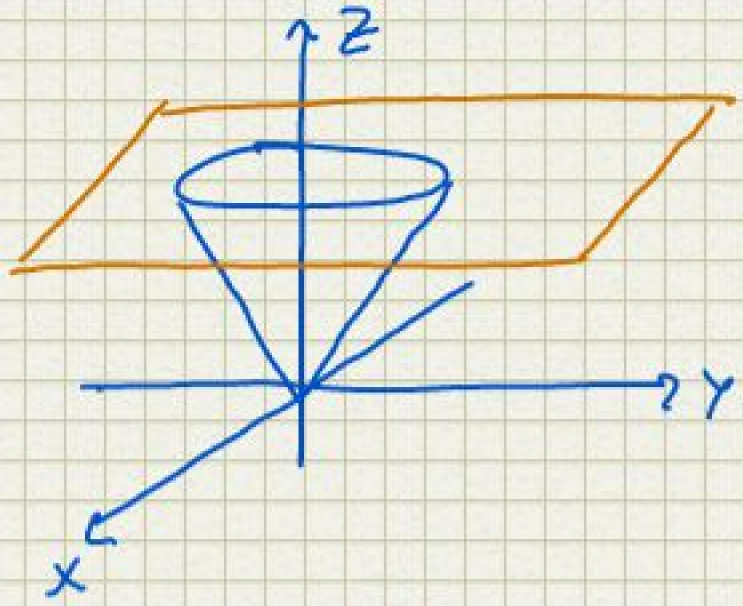
$$= \sqrt{2}\left(\pi + \frac{1}{4}\int_0^{2\pi}\left(1+\frac{1}{2}(1+\cos(2\theta))\right)d\theta\right)$$

$$= \sqrt{2}\left(\pi + \frac{3}{8}(2\pi)\right)$$

$\vec{F} = \langle x^2, y^2, z^2 \rangle$

$S: z = \sqrt{x^2+y^2}$

$\iint_S \vec{F} \cdot \vec{N}\, dS$



$S \Rightarrow -G = -z + \sqrt{x^2+y^2} = 0 \Rightarrow -\nabla G = \left\langle \frac{2x}{2\sqrt{x^2+y^2}}, \frac{2y}{2\sqrt{x^2+y^2}}, -1 \right\rangle$

* N is downward

$= \left\langle \frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}}, -1 \right\rangle$

$\begin{cases} z = 2 \\ z = \sqrt{x^2+y^2} \end{cases} \Rightarrow x^2+y^2 = 4$

$FI = \iint_D \langle x^2, y^2, (\sqrt{x^2+y^2})^2 \rangle \cdot \left\langle \frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}}, -1 \right\rangle dA$

HW Problems

2) $\vec{F} = \langle 10yz, 3xz, xy \rangle$

$\nabla \cdot \vec{F}$, $\nabla \times \vec{F}$, $\nabla \cdot (\nabla \times F)$

$\nabla \cdot \vec{F} = \frac{\partial}{\partial x}(10yz) + \frac{\partial}{\partial y}(3xz) + \frac{\partial}{\partial z}(xy) = 0$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 10yz & 3xz & xy \end{vmatrix} = \hat{i}(x-3x) - \hat{j}(y-10y) + \hat{k}(3z-10z)$$

$= \hat{i}(-2x) + \hat{j}(9y) - \hat{k}(7z)$

$\nabla \cdot (\nabla \times \vec{F}) = -2 + 9 - 7 = 0$

3) $\vec{F} = \langle e^{-x}\sin(y), e^{-x}\cos(y), 1 \rangle$

$\nabla \cdot F(1,3,-2) = \left(-e^{-x}\sin(y) - e^{-x}\sin(y)\right)\Big|_{(1,3,-2)}$

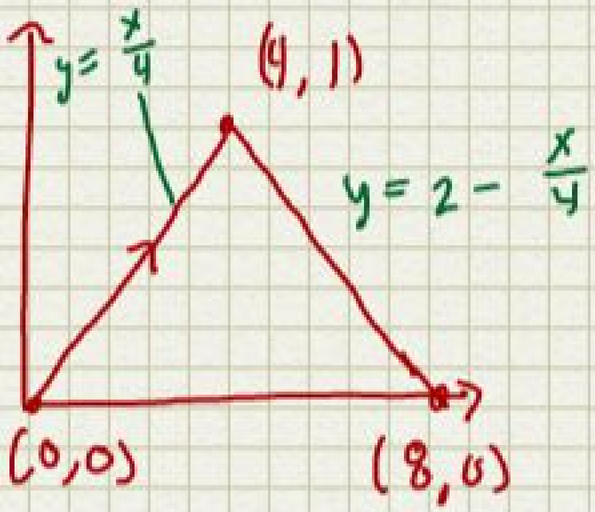
$= -2e^{-1}\sin(3)$

$\nabla \times F(1,3,-2) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ dx & dy & dz \\ e^{-x}\sin(y) & e^{-x}\cos(y) & 1 \end{vmatrix} = \hat{\imath}(0-0) - \hat{\jmath}(0-0) + \hat{k}\left(-e^{-x}\cos(y) - e^{-x}\cos(y)\right)\Big|_{(1,3,-2)}$

$= \langle 0, 0, -2e^{-1}\cos(3) \rangle$

7) $\int_C 4\,dy - dx$

$(0,0) \to (4,1) \to (8,0)$



$= \int_C \vec{F} \cdot d\vec{R} = \int_C \langle -1, 4 \rangle \cdot \langle dx, dy \rangle$

$= f(8,0) - f(0,0) = -8$

$f = \begin{cases} \int -1\,dx \\ \int 4\,dy \end{cases} = -x + 4y$

Check

easy way ~~use~~ conservative

$R_1 : \langle 4t, t \rangle \quad 0 \le x \le 4$

$0 \le 4t \le 4$

$0 \le t \le 1$

$R_2 : \langle 4t, 2-t \rangle$

$R_1' = \langle 4, 1 \rangle$

$R_2' = \langle 4, -1 \rangle$

$\int_{C_1} \vec{F} \cdot d\vec{R}_1 + \int_{C_2} \vec{F} \cdot d\vec{R}_2$

$= \int_0^1 \langle -1, 4 \rangle \cdot \langle 4, 1 \rangle\,dt + \int_1^2 \langle -1, 4 \rangle \cdot \langle 4, -1 \rangle\,dt$

$= \int_0^1 0\,dt + \int_1^2 -8\,dt = -8\int_1^2 dt = -8$

$- \vec{F}(x,y) = \langle -12x - 4y, -4x + 12y \rangle$

$\frac{\partial}{\partial y} = -4 \quad \frac{\partial}{\partial x} = -4$

$f = \int(-12x - 4y)\,dx = 6x^2 - 4xy \quad = \underline{-6x^2 - 4xy + 6y^2}$

$\int(-4x + 12y)\,dy = -4xy + 6y^2$

$- \vec{F}(x,y) = \langle -6y, 5x \rangle \ \underline{N}$

$- \vec{F}(x,y,z) = \langle -6x, -5y, 1 \rangle \ \underline{N}$

$- \vec{F} = \langle -6\sin(y), -8y - 6x\cos(y) \rangle$

$\partial y \qquad \partial x$

$-6\cos(y) = -6\cos(y)$

$f = \int -6\sin(y)\,dx \quad -6x\sin(y)$

$\int(-8y - 6x\cos(y))\,dy = -4y^2 - 6x\sin(y)$

$= \underline{-4y^2 - 6x\sin(y)}$

$- \vec{F}(x,y,z) = \langle -6x^2, -4y^2, 6z^2 \rangle \quad \nabla \cdot \vec{F} = 0$

$f = -2x^3, \ -\frac{4}{3}y^3, \ 2z^3 \qquad \underline{f = -2x^3 - \frac{4}{3}y^3 + 2z^2}$

$\oint_C y\,dx + 20x\,dy$ (not conservative)

$C: x^2+y^2=1$ Green's Theorm

$$= \iint_D \underbrace{(20x - y)}_{19}\,dA = 19\iint_D dA = 19\pi(1)^2 = 19\pi$$

4) $\operatorname{div}\vec{F}$ $\vec{F} = \nabla f = \langle \partial_x f, \partial_y f, \partial z f \rangle$

$f = xy^3z^2$ $= \langle y^3z^2, 3xy^2z^2, 2xy^3z \rangle$

$\operatorname{div}\vec{F} = \nabla$

$= \underline{0 + 6xyz^2 + 2xy^3}$

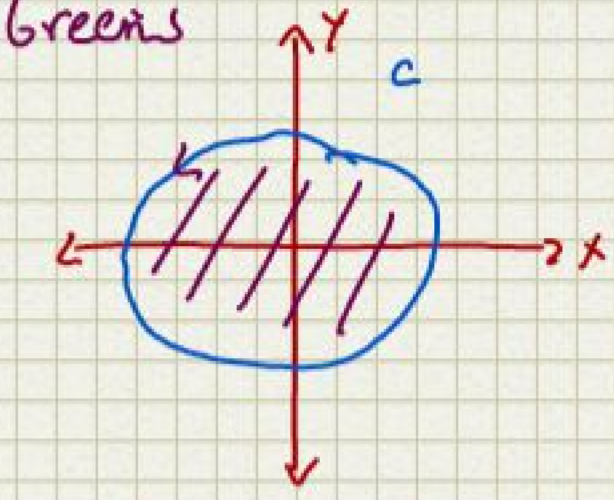
$\nabla\nabla f = \nabla f = \partial_{xx} f + \partial_{yy} f + \partial_{zz} f$

4/22/2014

13.6

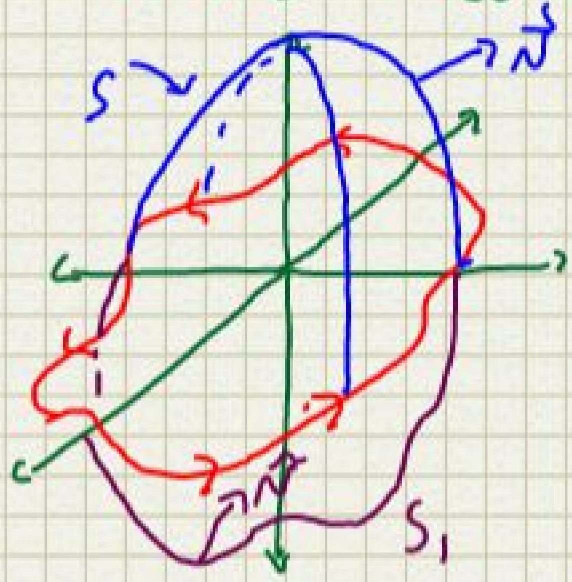
Stoke's Theorm in 3D

Greens



$$\oint_C \vec{F}\cdot d\vec{R} = \iint_D (f_{2x} - f_{1y})\, dA$$

Stokes' Theorm

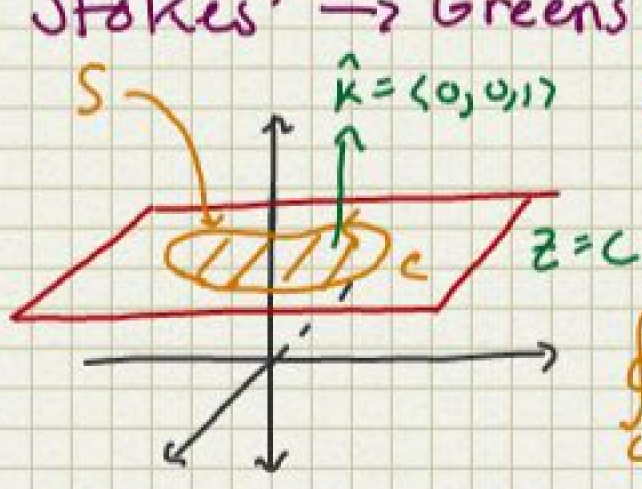


* no canterclockwise or clockwise

$$\oint_C \vec{F}\cdot d\vec{R} = \iint_S (\nabla \times \vec{F})\cdot \hat{n}\, dS$$

$$= \iint_{S_1} (\nabla \times \vec{F})\cdot \hat{N}\, dS$$

Stokes' → Green's



$$\vec{F} = \langle f_1(x,y,z), f_2(x,y,z), f_3(x,y,z)\rangle$$

$$= \langle f_1(x,y,c), f_2(x,y,c), f_3(x,y,c)\rangle$$

$$\oint_C \vec{F}\cdot d\vec{R} = \iint_S (\nabla\times\vec{F})\cdot\langle 0,0,1\rangle\, dA$$

$$\nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial x & \partial y & \partial z \\ f_1 & f_2 & f_3 \end{vmatrix} = (\quad)\hat{i} - \hat{j}(\quad) + \hat{k}\left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right)$$

$$\oint_C \vec{F}\cdot d\vec{R} = \iint_{D,\ z=c} \left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right) dA$$

Ex

$$\oint_C \vec{F}\cdot d\vec{R} = \iint_S (\nabla\times\vec{F})\cdot \hat{N}\, ds$$

$$\oint_C \frac{1}{2}y^2 dx + z\,dy + x\,dz \qquad C:\begin{cases} x+z=1 \\ x^2+2y^2+z^2=1 \end{cases}$$

* orientated CCW when viewed from above

$$= \oint_C \langle \tfrac{1}{2}y^2, z, x\rangle\, d\vec{R}$$

$$\oint \vec{F}\cdot d\vec{R} = \iint_S \underbrace{(\nabla\times\vec{F})}_{\vec{G}}\cdot\hat{N}\, ds = \iint_D G(x,y,z=1-x)\cdot\langle -f_x, -f_y, 1\rangle\, dA$$

$$G = \nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ \frac{1}{2}y^2 & z & x \end{vmatrix} = \hat{i}(0-1) - \hat{j}(1-0) + \hat{k}(-y)$$
$$= \langle 1, 1, -y\rangle$$

$$f = z = 1-x \qquad \langle -f_x, -f_y, 1\rangle = \langle 1, 0, 1\rangle$$

$$\iint_D \langle -1, 1, y\rangle\cdot\langle 1, 0, 1\rangle\, dA$$

$$= \iint_D (-1-y)\, dA = \int_0^{\pi}\int_0^{\cos(\theta)} (-1 - r\sin(\theta))\, r\, dr\, d\theta$$

$$= \int_0^{\pi} -\frac{r^2}{2} - \frac{r^3}{3}\sin(\theta)\Big|_0^{\cos(\theta)} d\theta = \int_0^{\pi} -\frac{1}{2}\cos^2(\theta) - \frac{1}{3}\cos^3\theta\sin\theta\, d\theta$$

$$u = \cos(\theta)$$

$$= -\frac{1}{2}\int_0^{\pi} \frac{1}{2}(1+\cos(\theta))\, d\theta + \frac{1}{3}\int_{-}^{-} u^3\, du$$

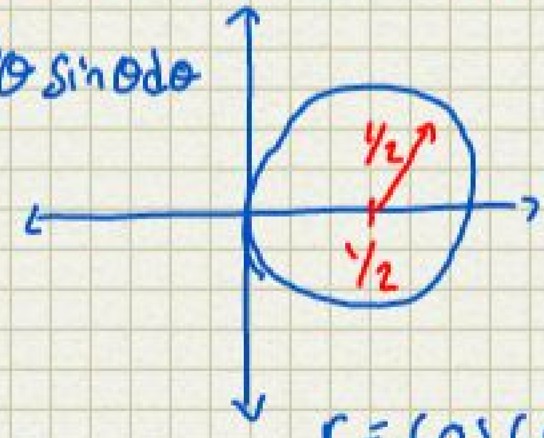
$$= -\frac{1}{4}\pi + \frac{1}{3}\,\frac{\cos(\theta)}{4}\Big|_0^{\pi} = -\frac{1}{4}\pi + \frac{(1-1)}{\sim} = \frac{-1}{4\pi}$$

$$C\begin{cases} x+z=1 \\ x^2+2y^2+z^2=1 \end{cases}$$

$z = 1-x$

$x^2 - x + y^2 = 0$

(circle at $x = \frac{1}{2}, r = \frac{1}{2}$)

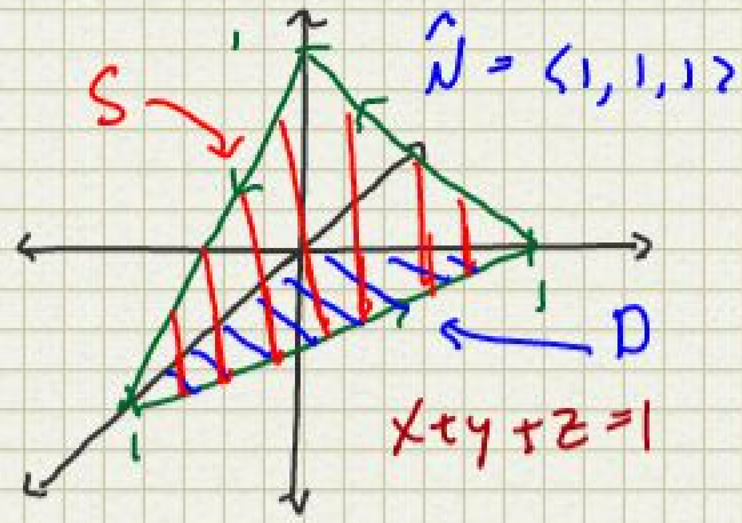


$r = \cos(\theta)$

$0 \le \theta \le \pi$

* for Stokes' theorm use the plane that will make it easy to solve with

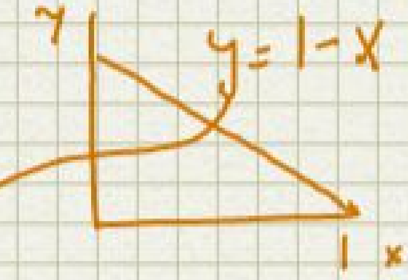
$$\oint_C \vec{F}\cdot d\vec{R} = \iint_S (\nabla\times\vec{F})\cdot\hat{N}\,ds$$



$$\vec{F} = -\frac{3}{2}y^2\hat{\imath} - 2xy\,\hat{\jmath} + yz\hat{k}$$

$$(\nabla\times\vec{F}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ -\frac{3}{2}y^2 & -2xy & yz \end{vmatrix} = \hat{\imath}(z-0) - \hat{\jmath}(0-0) + \hat{k}(-2y-3y)$$

$$= \langle z, 0, -y\rangle$$



$$\oint_C \vec{F}\cdot d\vec{R} = \iint_D \langle 1-x-y, 0, -y\rangle\cdot\langle 1,1,1\rangle\,dA$$

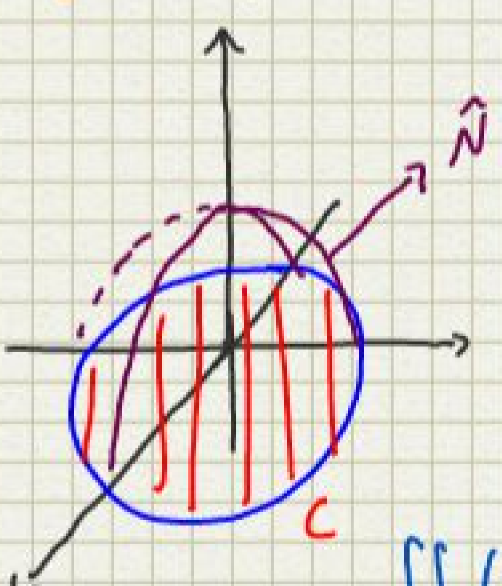
$$= \iint_D 1-x-y+y\,dA = \iint_D (1-x)\,dA = \int_0^1\int_0^{1-x}(1-x)\,dy\,dx$$

$$= \int_0^1 (1-x)^2\,dx = -\frac{(1-x)^3}{3}\Big|_0^1 = -\frac{1}{3}$$

Ex

$\iint_S (\nabla\times\vec{F})\cdot\vec{N}\,ds \quad \vec{F} = \langle x, y, ze^{xy}\rangle \quad z = 1-x^2-2y^2$

$z \geq 0$ upward normal



$\hat{k} = \langle 0,0,1\rangle$
on surface
$z=0$

$$\nabla\times\vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ x & y & ze^{xy} \end{vmatrix}$$

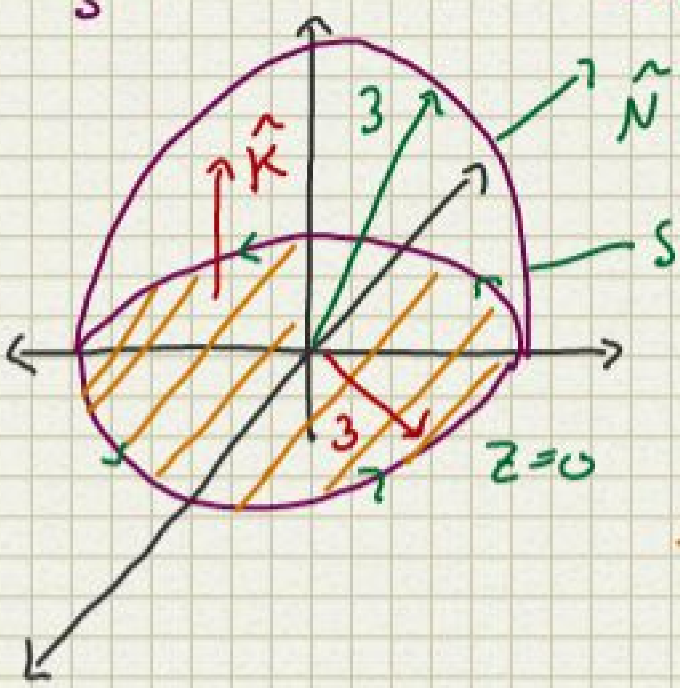
$$= \hat{\imath}(---) - \hat{\jmath}(---) + (0)\hat{k}$$

$$\iint_{S\ z=0} (\nabla\times\vec{F})\cdot\hat{N} = \iint(\hat{\imath}(--) - \hat{\jmath}(--) + 0\hat{k})\cdot\langle 0,0,1\rangle\,dA$$

$$= 0$$

$\vec{F} = \langle z, 2x, 3y \rangle$

$\iint_S (\nabla \times \vec{F}) \cdot \hat{N}\, ds$ upper semisphere $z = \sqrt{9 - x^2 - y^2}$ oriented upward



$$\oint_C \vec{F} \cdot d\vec{R} = \iint_{\substack{D \\ z=0}} (\nabla \times \vec{F}) \cdot \hat{k}\, dA$$

$$\nabla \times \vec{F} = \langle - - - \rangle \hat{\imath} - \langle - - - \rangle \hat{\jmath} + \hat{k}(2 - 0)$$

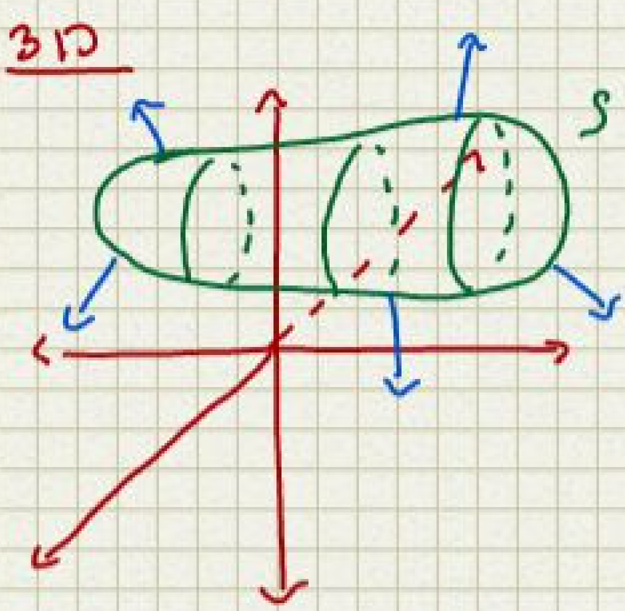
$$= \iint_D \langle ---, ---, 2 \rangle \cdot \langle 0, 0, 1 \rangle\, dA$$

$$= 2 \iint_D dA = 2 \cdot \pi 3^2 = 18\pi$$

4/23/2014

13.7 Divergence Theorm

3D



applies to whole closed surface

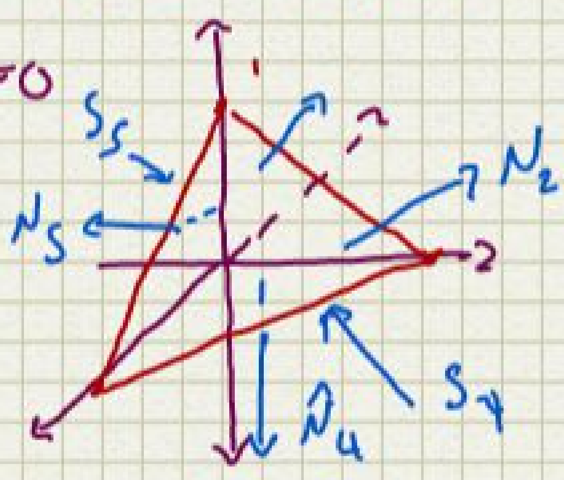
$\hat{N}$ is the outward normal

$$\iint_S \vec{F}\cdot\hat{N}\,ds = \iiint_{\forall} \nabla\cdot\vec{F}\,d\forall$$

$\iint_S \vec{F}\cdot\hat{N}\,ds \quad \vec{F} = \langle x^2, xy, x^3y^3\rangle$

S is the surface that bounds the tetrahedron

$x+y+z=1 \quad x=0,\ y=0,\ z=0$



$$\iint_{S_1} \vec{F}\cdot\hat{N}_1\,ds + \iint_{S_2} \vec{F}\cdot\hat{N}_2\,ds + \iint_{S_3} \vec{F}\cdot\hat{N}_3\,ds + \iint_{S_4} \vec{F}\cdot\hat{N}_4\,ds$$

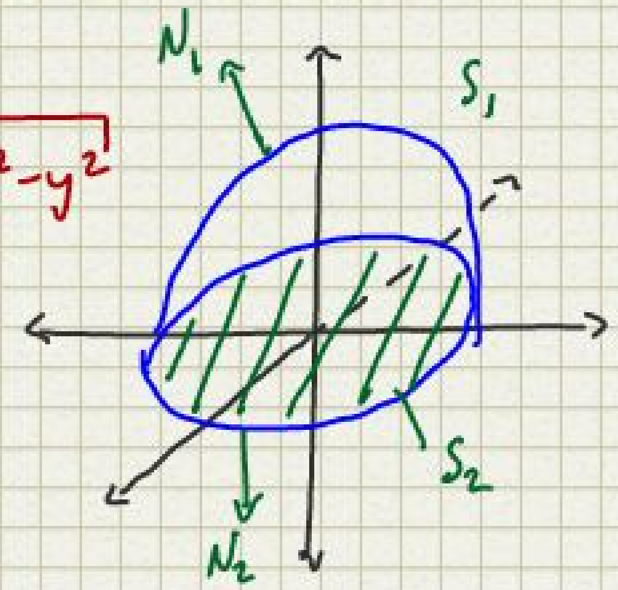
$$= \iiint_{\forall} (2x+x+0)\,d\forall = \int_0^1\int_0^{1-x}\int_0^{z=1-x-y} 3x\,dz\,dy\,dz$$

$$= \int_0^1\int_0^{1-x} 3x(1-x-y)\,dy\,dx = \int_0^1 3x\left((1-x)y - \frac{1}{2}y^2\right)\Big|_0^{1-x} dx$$

$$= \int_0^1 3x\left((1-x)^2 - \frac{1}{2}(1-x)^2\right) dx = \int_0^1 3x\,\frac{(1-x)^2}{2}\,dx$$

$$= \frac{3}{2}\int_0^1 x - 2x^2 + x^3\,dx = \frac{3}{2}\left(\frac{x^2}{2} - \frac{2}{3}x^3 + \frac{1}{4}x^4\right)\Big|_0^1 = \frac{1}{8}$$

Divergence Theorm



$\vec{F} = \langle 2x, -3y, 5z \rangle \qquad S \begin{cases} z = \sqrt{9-x^2-y^2} \\ z = 0 \end{cases}$

$$\iint_S \vec{F} \cdot \underset{\text{outward}}{\underset{\uparrow}{\hat{N}}} \, ds = \iiint_{\forall} \nabla \cdot \vec{F} \, d\forall$$

$$\iint_S \vec{F} \cdot \hat{N} \, ds = \overbrace{\iint_{S_1} \vec{F} \cdot \hat{N}_1 \, ds}^{I_1} + \overbrace{\iint_{S_2} \vec{F} \cdot \hat{N}_2 \, ds}^{I_2}$$

$$I_1 = \iint_D \langle 2x, -3y, 5\sqrt{9-x^2+y^2} \rangle \cdot \langle \overset{f_x}{\frac{x}{\sqrt{9-x^2-y^2}}}, \overset{f_y}{\frac{y}{\sqrt{9-x^2-y^2}}}, \overset{1}{1} \rangle \, dA$$

$$= \iint_D \left(\frac{2x^2}{\sqrt{9-x^2-y^2}} - \frac{3y^2}{\sqrt{9-x^2-y^2}} + 5\sqrt{9-x^2-y^2} \right) dA$$

$$= \int_0^{2\pi} \int_0^3 \left(\frac{2r^2 \cos(\theta)}{\sqrt{9-r^2}} - \frac{3r^2 \sin(\theta)}{\sqrt{9-r^2}} + 5\sqrt{9-r^2} \right)$$

$u = 9 - r^2 \quad du = -2r \, dr$

$$= \int_0^{2\pi} \int \left(2\cos^2(\theta) \frac{9-u}{\sqrt{u}} - 3\sin^2(\theta) \frac{9-u}{\sqrt{u}} + 5\sqrt{u} \right) \cdot -\frac{du}{2} \, d\theta$$

$$= \int_0^{2\pi} -\frac{1}{2} \left[\left(-2\cos^2(\theta) + 3\sin^2(\theta) + 5 \right) \frac{2}{3} u^{3/2} + \left(2\cos^2(\theta) - 3\sin^2(\theta) \right) 9 \cdot 2\sqrt{u} \right] d\theta$$

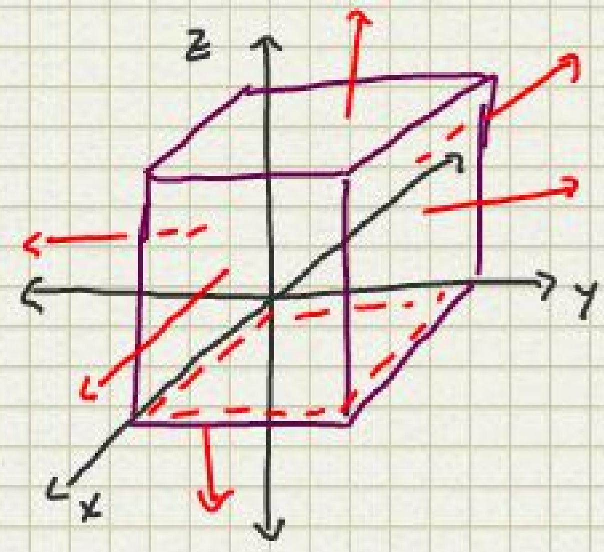
$= 72\pi$

$$I_2 = \iint_D \langle 2x, -3y, 0 \rangle \cdot \langle 0, 0, -1 \rangle \, dA = 0$$

$$\iiint_{\forall} (2 - 3 + 5) \, d\forall = 4 \iiint_{\forall} d\forall = 2 \cdot \frac{4}{3} \pi (3)^3 = 72\pi$$

open surface but surface is the union of many surfaces

$$\vec{F} = \langle xy, 0, -z^2 \rangle \qquad \iint_S \vec{F} \cdot \hat{N}\, ds$$



$$\iint_s \vec{F} \cdot \hat{N}\, ds = \iint_{\underbrace{s_1 + s_2 + s_3 + s_4 + s_5 + s_6}_{\nabla \cdot \vec{F}} - s_6} \vec{F} \cdot \hat{N}\, ds$$

$$\iiint_{\forall} \nabla \cdot \vec{F}\, ds - \iint_{S_6} \vec{F} \cdot \hat{N}_6\, dS$$

$$= \iiint_{\forall} (y + 0 - 2z)\, d\forall - \overbrace{\iint_{S_6} \langle xy, 0, 0 \rangle \cdot \langle 0, 0, -1 \rangle\, dA}^{0}$$

$$= \int_0^1 dx \int_0^1 y\, dy \int_0^1 dz - 2 \int_0^1 dx \int_0^1 dy \int_0^1 z\, ds$$