4/24/2014

2) $\iint_S (x^2+y^2)\,ds \quad S: \; z = xy$

g

$\iint (x^2+y^2)\sqrt{1+y^2+x^2}\,dA$

$\int_0^{\pi/2}\int_0^2 R^2\sqrt{1+R^2}\,r\,dr\,d\theta$

$u = 1+R^2 \quad R^2 = u-1$

$du = 2R\,dr$

$SI = \frac{1}{2}\int_0^{\pi/2}\int_{---}^{---} (u^{3/2} - u^{1/2})\,du = \frac{1}{2}\int_0^{\pi/2} \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\Big|\,d\theta$

$= \frac{1}{2}\int_0^{\pi/2} \frac{2}{5}(1+R^2)^{5/2} - \frac{2}{3}(1+R^2)^{3/2}\Big|_0^2\,d\theta$

5) $\iint_S (x^2+y^2)\,ds \quad S: x^2+y^2+z^2 = 4 \quad z \geq 0$

$ds = \frac{r}{z}dA = \frac{2}{\sqrt{4-x^2-y^2}}\,dA$

$SI = \iint_D (x^2+y^2)\frac{2}{\sqrt{4-x^2-y^2}}\,dA = \int_0^{2\pi}\int_0^2 R^2 \frac{1}{\sqrt{4-R^2}}\,r\,dr\,d\theta$

$u = 4-R^2 \quad R^2 = 4-u$

$du = -2R$

$= \int_0^{2\pi}\int_{---}^{---} \frac{4-u}{\sqrt{u}}(-)\frac{du}{2}$

$= -\frac{1}{2}\int_0^{2\pi}\int_{---}^{---} (4u^{-1/2} - u^{1/2})\,du$

flux integral

6) $F = x\hat{i} + 2y\hat{j} + 2\hat{k}$ $S: \begin{cases} x+2y+z=1 \\ x\geq 0 \;\; y\geq 0 \;\; z\geq 0 \end{cases}$ $\hat{N}$ upward

$$\iint_S \vec{F}\cdot\hat{N}\,ds = \iint_D \langle x, y, 1-x-2y\rangle \langle 1,2,1\rangle\, dA$$

$$= \iint_D (x+2y+1-x-2y)\,dA = \iint_D dA = \frac{1\cdot\frac{1}{2}}{2} = \frac{1}{4}$$

8

$$\oint_C x^2y^2dx + dy + z^2dz \qquad x^2+y^2=1 \quad z=1$$

$$\oint \vec{F}\cdot d\vec{R} \quad \vec{F} = \langle x^2y^2, 1, z^2\rangle$$

$$= \iint_S \nabla\times\vec{F}\cdot\hat{N}\,ds$$

$$(\nabla\times\vec{F}) = (\_\,\_\,\_)\hat{i} - (\_\,\_\,\_)\hat{j} + (0-2yx^3)\hat{k}$$

$$\iint_S \nabla\times\vec{F}\cdot\hat{N}\,ds = \iint_D \langle --, --, -2yx^3\rangle\langle 0,0,-1\rangle$$

$$= \iint_D 2yx^3\,dA = 2\int_0^{2\pi}\int_0^1 r\sin(\theta)\, r^3\cos^3\theta\, r\,dr\,d\theta$$

$$= 2\int_0^{2\pi}\sin(\theta)\cos^3(\theta)\,d\theta \int_0^1 r^5\,dr = 2\left(-\frac{u^4}{4}\Big|_{--}^{--}\right)\left(\frac{r^6}{6}\Big|_0^1\right)$$

$u = \cos(\theta)$

$du = -\sin(\theta)\,d\theta$

$$= 2\left(-\cos^4(\theta)\Big|_0^{2\pi}\right)\left(\frac{1}{6}\right)$$

10) $\iint_S \nabla \times \vec{F} \cdot \vec{N}\, ds = \iint_{S_1} \nabla \times \vec{F} \cdot \hat{N}\, ds$

$\vec{F} = \langle x, x, xyz \rangle$ $\quad S: z = 4 - x^2 - y^2 \quad z \geq 0$

Curl

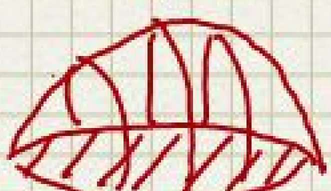
$\nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial x & \partial y & \partial z \\ x & y & xyz \end{vmatrix} = (\_\_)\hat{i} - (\_\_)\hat{j} + (0)\hat{k}$

17) Divergence Theorm $\quad$ find volume inside hemisphere

$\oiint_S \vec{F} \cdot \hat{N}\, ds \qquad \vec{F} = \langle 10xy^2z, 10yx^2z, z^2 \rangle$

$= \iiint_V \nabla \cdot \vec{F}\, dV \qquad S: x^2 + y^2 + z^2 = 1$

$x^2 + y^2 \leq 1 \quad z = 0$



$\nabla \cdot \vec{F} = (10y^2z + 10x^2z + 2z)$

$I = \iiint_V (10z(x^2 + y^2) + 2z\, dV$

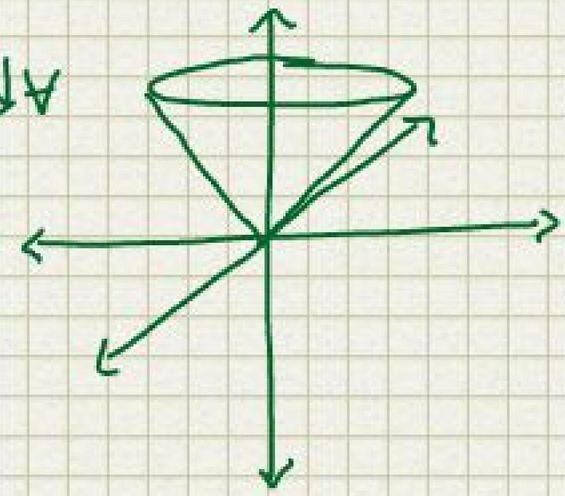
$= \int_0^{2\pi} \int_0^{\pi/2} \int_0^1 (10\rho\cos(\phi)(\rho^2\sin^2(\phi) + 2\rho\cos(\phi))\rho^2 \sin(\phi)\, d\rho\, d\phi\, d\theta$

$= 10\int_0^{2\pi} d\theta \int_0^{\pi/2} \sin^3(\phi)\cos(\phi)\, d\phi \int_0^1 \rho^5\, d\rho + 2\int_0^{2\pi} d\theta \int_0^{\pi/2} \sin(\phi)\cos(\phi)\, d\phi \int_0^1 \rho^3\, d\rho$

$= 10(2\pi)\frac{1}{4}\sin^4(\phi)\Big|_0^{\pi/2}\left(\frac{1}{6}\right) + 2(2\pi)\frac{\sin^2(\phi)}{2}\Big|_0^{\pi/2}\left(\frac{1}{4}\right)$

$\frac{5}{6}\pi + \frac{1}{2}\pi$

$$\oiint_S \vec{F}\cdot\vec{N}\,dS = \iiint_{\forall} (y^2+z^2+x^2)\,d\forall$$



$$\vec{F} = xy^2\hat{\imath} + yz^2\hat{\jmath} + zx^2\hat{k}$$

$$S = \begin{cases} \rho = 2 \text{ above} \\ \phi = \pi/4 \text{ below} \end{cases}$$

$$\int_0^{2\pi}\int_0^{\pi/4}\int_0^{2} \rho^2\ \rho^2 \sin(\phi)\,d\rho\,d\phi\,d\theta$$

$$= \int_0^{2\pi} d\theta \int_0^{\pi/4} \sin(\phi)\,d\phi \int_0^{2} \rho^4\,d\rho$$

$$= 2\pi\left(-\cos(\phi)\Big|_0^{\pi/4}\right) \tfrac{1}{5}\rho^5\Big|_0^2 = 2\pi\left(-\tfrac{\sqrt{2}}{2}+1\right)\tfrac{32}{5}$$

25

$\vec{F} = \langle y\sin(z), x\sin(z)+2y, xy\cos(z)$

Show vector field is conservative

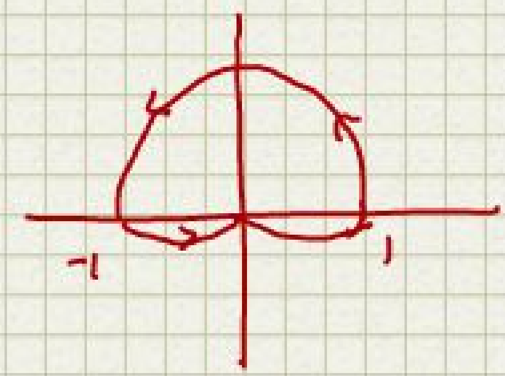
$$\nabla\times\vec{F} = \vec{0} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ d_x & d_y & d_z \\ y\sin(z) & x\sin(z)+2y & xy\cos(z) \end{vmatrix}$$

$$= \hat{\imath}(x\cos(z) - x\cos(z)) - \hat{\jmath}(y\cos(z) - y\cos(z)$$
$$+ \hat{k}(\sin(z) - \sin(z) = \vec{0}$$

$$f = \begin{cases} xy\sin(z) \\ xy\sin(z) + y^2 \\ xy\sin(z) \end{cases} \qquad f = xy\sin(z) + y^2$$

$$I = \oint_C (3y\,dx - 2x\,dy) \qquad C: 1+\sin(\theta)$$

$$\langle 3y, -2x \rangle$$



$$= \iint_D (f_{2x} - f_{1y})\,dA$$

$$= \iint_D (-2-3)\,dA$$

$$= -5\int_0^{2\pi}\int_0^{1+\sin(\theta)} r\,dr\,d\theta = -5\int_0^{2\pi} \frac{(1+\sin\theta)^2}{2}\,d\theta$$

$$= -\frac{5}{2}\int_0^{2\pi} 1 + 2\sin(\theta) + \sin^2(\theta)\,d\theta = \frac{-15}{2}\pi$$

Final 12th Monday 4:30-7pm

Chem 113

- bring orange scantron
- 1 blue book
- Id
- No calc
- Test's sold in math building
  - Spring 2013 structure