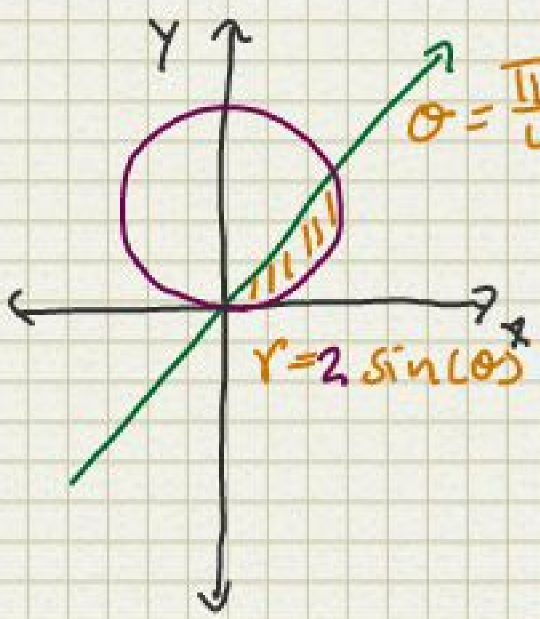
3/13/2014

$$\iint_{P(x,y)} f(x,y)\,dA = \iint_{D(\theta,R)} f(r\cos(\theta), r\sin(\theta))\, r\,dr\,d\theta$$

$$= \int_{\theta_1}^{\theta_2}\int_{R_1(\theta)}^{R_2(\theta)} f(r,\theta)\; r\,dr\,d\theta$$

compute the area below $y=x$ and above $x^2+y^2-2y=0$



$x^2+y^2-2y=0$

$x^2+(y^2-2y+1)=1$

$x^2+(y-1)^2=1$

$*\ \sin^2(\theta)=\frac{1}{2}(1-\cos(2\theta))$

$\cos^2(\theta)=\frac{1}{2}(1+\cos(2\theta))$

Line at x-axis

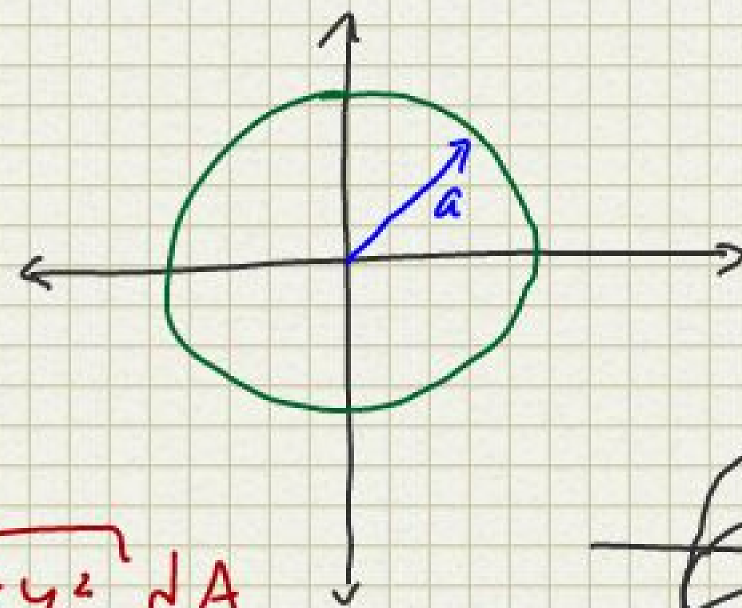
$$\int_{\theta=0}^{\theta=\pi/4}\int_{r=0}^{r=2\sin(\theta)} r\,dr\,d\theta = \int_0^{\pi/4}\frac{r^2}{2}\Big|_0^{2\sin(\theta)}d\theta = \int_0^{\pi/4}\frac{4\sin^2(\theta)}{2}\,d\theta$$

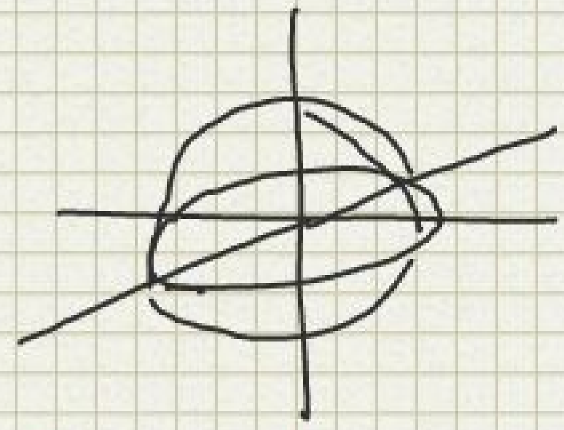
$$= \int_0^{\pi/4} 2\sin^2(\theta)\,d\theta = \int_0^{\pi/4}(1-\cos(2\theta))\,d\theta = \theta-\frac{\sin(2\theta)}{2}\Big|_0^{\pi/4}$$

$$= \frac{\pi}{4}-\frac{\sin(\pi/2)}{2} = \frac{\pi}{4}-\frac{1}{2}$$

Sphere

$$x^2+y^2+z^2=a^2$$





$$z=\pm\sqrt{a^2-x^2-y^2}$$

$$z=\sqrt{a^2-x^2-y^2}\geq 0$$

$$\text{Volume}=2\iint_D \sqrt{a^2-x^2-y^2}\,dA$$

$$=2\int_0^{2\pi}\int_0^a \sqrt{a^2-r^2}\,r\,dr\,d\theta = 2\int_0^{2\pi}\int \sqrt{u}\,\frac{du}{-2}\,dr$$

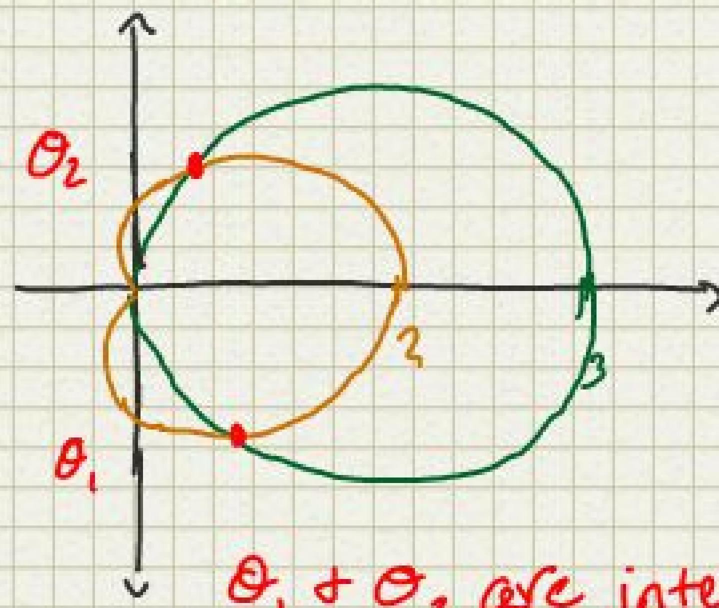
$$u=a^2-r^2$$

$$\frac{du}{-2}=-2r\,dr$$

$$=2\int_0^{2\pi} -\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}u^{3/2}\Big|\,d\theta = 2\int_0^{2\pi} -\frac{1}{3}(a^2-r^2)^{3/2}\Big|_0^a$$

$$=\frac{-2}{3}(0-a^2)^{3/2}\int_0^{2\pi} d\theta = \frac{2}{3}a^3\Big|_0^{2\pi} = \frac{4}{3}\pi a^3$$

$$\iint_D \frac{1}{x}\, dA$$



θ_1 + θ_2 are intersection points

D: in $R = 3\cos(\theta)$

out $R = 1 + \cos(\theta)$

$$= \int_{\theta_1}^{\theta_2} \int_{1+\cos\theta}^{3\cos(\theta)} \frac{1}{r\cos(\theta)}\, r\, dr\, d\theta$$

$$\begin{cases} r = 3\cos(\theta) \\ r = 1 + \cos(\theta) \end{cases} \Rightarrow \begin{cases} r = 3\cos(\theta) \\ 3\cos(\theta) = 1 + \cos(\theta) \end{cases}$$

$$\begin{cases} r = 3\cos(\theta) \\ 2\cos(\theta) = \frac{1}{2} \end{cases} \qquad 2 = 3\cos(\theta)$$

$$\theta = \cos^{-1}\left(\tfrac{1}{2}\right) \to 60^\circ,\ -60^\circ$$

$$\int_{-\pi/3}^{\pi/3} \int_{1+\cos(\theta)}^{3\cos(\theta)} \frac{1}{r\cos(\theta)}\, r\, dr\, d\theta = \int_{-\pi/3}^{\pi/3} \frac{1}{\cos(\theta)}\, r \Big|_{1+\cos(\theta)}^{3\cos(\theta)}\, d\theta$$

$$= \int_{-\pi/3}^{\pi/3} \frac{1}{\cos(\theta)} \left(3\cos(\theta) - 1 - \cos(\theta)\right) d\theta$$

$$= \int_{-\pi/3}^{\pi/3} (2 - \sec(\theta)\, d\theta = 2\theta - \ln|\sec(\theta) + \tan(\theta)|\Big|_{-\pi/3}^{\pi/3}$$

$$= \ln|\sec(-\pi/3) + \tan(-\pi/3)$$

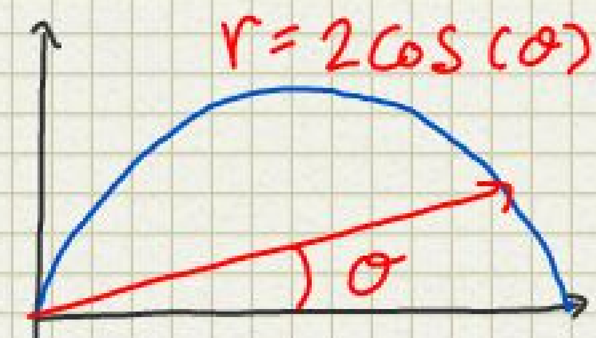
$$\int_0^2\int_0^{\sqrt{2x-x^2}} y\sqrt{x^2+y^2}\,dy\,dx$$

$y=\sqrt{2x-x^2}$

$y^2=2x-x^2$

$(x^2-2x+1)+y^2=1$

$(x-1)^2+y^2=1$



$$\int_0^{\pi/2}\int_{r=0}^{r=2\cos(\theta)} r\sin(\theta)\sqrt{r^4}\; r\,dr\,d\theta$$

$\sin(\theta)\,r^3$

$u=\cos(\theta)$

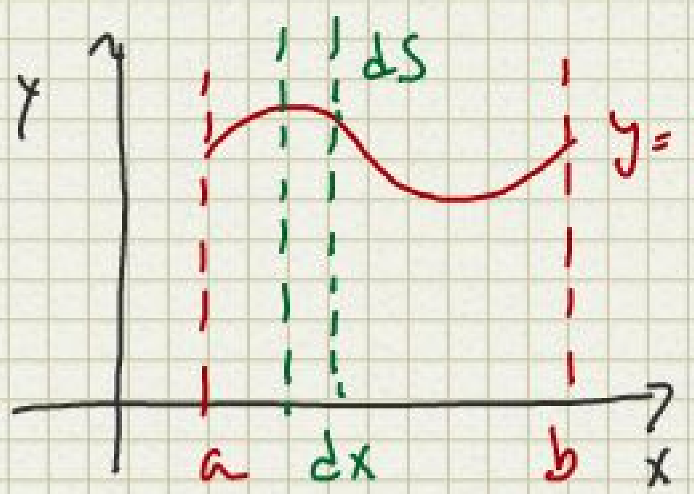
$du=-\sin(\theta)\,d\theta$

$$=\int_0^{\pi/2}\sin(\theta)\left.\frac{r^4}{4}\right|_0^{2\cos(\theta)} d\theta=\int_0^{\pi/2}\sin(\theta)\left(4\cos^4(\theta)-0\right)d\theta$$

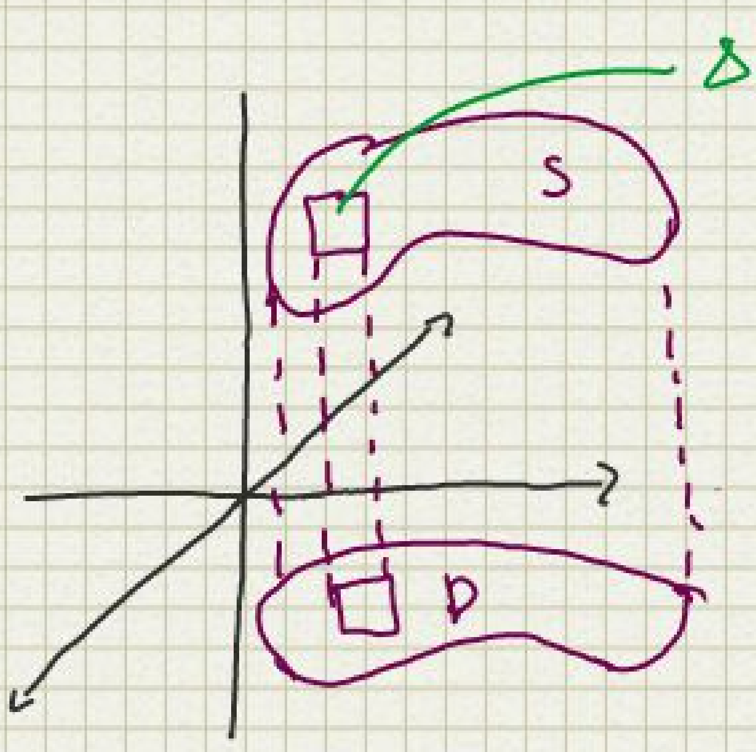
$$=4\int_{---}^{---}-u^4\,du=-4\left.\frac{u^5}{5}\right|_{---}^{---}=\left(-\frac{4}{5}\right)\cos^5(\theta)\Big|_0^{\pi/2}$$

$$=-\frac{4}{5}\left(0-1^5\right)=\frac{4}{5}$$

12.4 surface area



$$S = \int_S ds = \int_a^b \sqrt{1+(f'(x))^2}\, dx$$



$$dS = dA\sqrt{1+f_x^2+f_y^2}$$

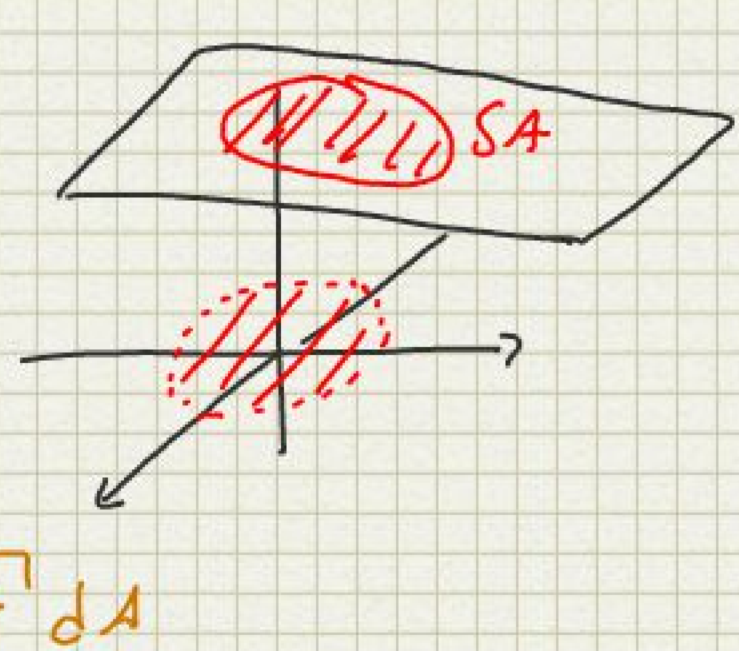
$$SA = \iint_S ds = \iint_D \sqrt{1+f_x^2+f_y^2}\, dA$$

Ex

$x+y+z=4$

Inside

$x^2+y^2 \le 4$



$$S = \iint_S ds$$

$$= \iint \sqrt{1+(-1)^2+(-1)^2}\, dA$$

$z = 4-x-y$

$z_x = -1$

$z_y = -1$

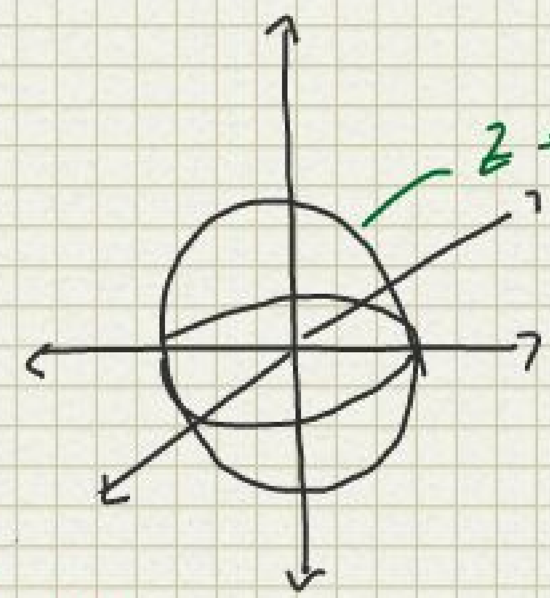
$$\sqrt{3}\iint_D dA$$

$$= \sqrt{3}\,\pi(2)^2$$

$$= 4\sqrt{3}\pi$$

surface area sphere $R = a$

$$x^2 + y^2 + z^2 = a^2$$



$$SA = 2\iint_D \sqrt{1 + z_x^2 + z_y^2}\, dA$$

$$\frac{\partial}{\partial x}(\quad) = 2x + 0 + 2z z_x$$

$$z_x = -\frac{x}{z} \quad z_y = -\frac{y}{z}$$

$$2\iint_D \sqrt{1 + z_x^2 + z_y^2}\, dA = 2\iint_D \sqrt{1 + \frac{x^2}{z^2} + \frac{y^2}{z^2}}\, dA$$

$$= 2\iint_D \sqrt{\frac{\overbrace{z^2 + x^2 + y^2}^{\to a^2}}{z^2}}\, dA = 2\iint_D \frac{a}{z}\, dA = 2\iint_D \frac{a}{\sqrt{a^2 - x^2 - y^2}}\, dA$$

Polar Coordinates

$$= 2\int_0^{2\pi}\int_0^a \frac{a^2}{\sqrt{a^2 - r^2}}\, r\,dr\,d\theta = 2\int_0^{2\pi}\int_{---}^{---} \frac{a}{\sqrt{u}}\,\frac{du}{-2}\,d\theta$$

$$u = 2 - r^2$$
$$-2r\,dr = du$$

$$= -\int_a^{2\pi} a\, 2u^{1/2} \Big|_{---}^{---} d\theta = -2a\int_0^{2\pi} (a^2 - r^2)^{1/2}\Big|_0^a d\theta$$

$$= 2a\left(0^{1/2} + (a^2)^{1/2}\right)\Big|_0^{2\pi} d\theta = 4\pi a^2$$