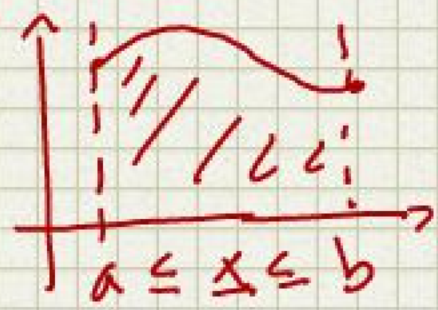
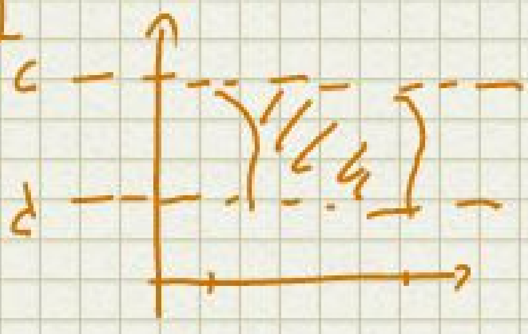
12.2

Type I



Type II



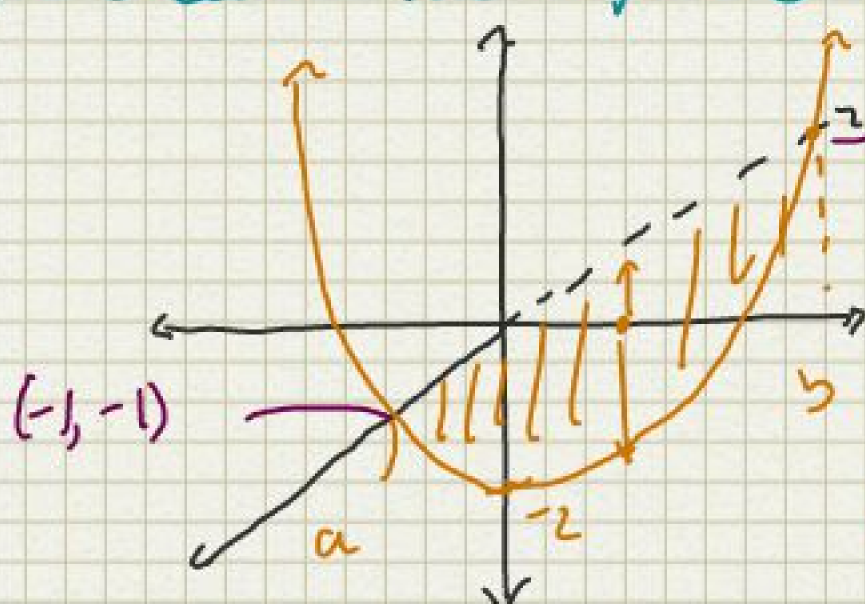
$$\iint_D f(x,y)\,dA \;\xrightarrow{\text{Type I}}\; \int_a^b \int_{y_1}^{y_2} f(x,y)\,dy\,dx$$

$$\iint_D f(x,y)\,dA \;\xrightarrow{\text{Type II}}\; \int_c^d \int_{x_1(y)}^{x_2(y)} f(x,y)\,dx,\,dy$$

$f(x,y) \ge 0$ for all $(x,y) \in D$

$\iint_D f(x,y)\,dA$ = volume $\qquad \iint_D dA$ - area (D)

Find the area of the region D in between $y = x$, $y = x^2 - 2$



$$\begin{cases} y = x \\ y = x^2 - 2 \end{cases}$$

$$\begin{cases} y = x \\ x^2 - x - 2 = 0 \end{cases}$$

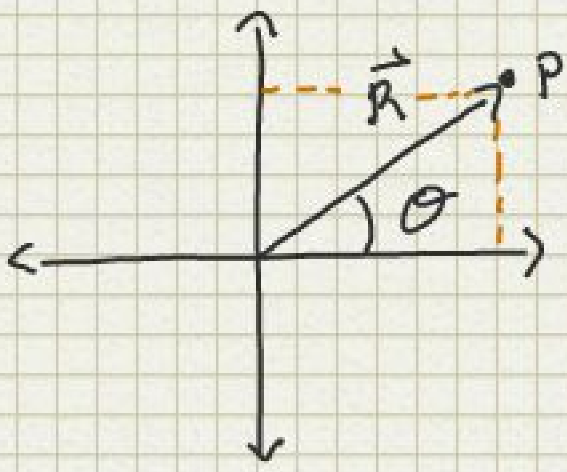
$y = x$

$$x_{1,2} = \frac{1 \pm \sqrt{(-1)^2 + 8}}{2} = \Big\langle \begin{matrix} 2 \\ -1 \end{matrix}$$

$$\text{Area} = \iint_D 1\,dA = \int_{-1}^{2}\int_{x^2-2}^{x} 1\,dy\,dx = \int_{-1}^{2} y\,\Big|_{x^2-2}^{x} dx = \int_{-1}^{2} x - (x^2 - 2)\,dx =$$

$$= \frac{x^2}{2} - \frac{x^3}{3} + 2x\,\Big|_{-1}^{2} = \left(4 - \frac{9}{3} + 4 - \frac{1}{2}\right) = \frac{9}{2}$$

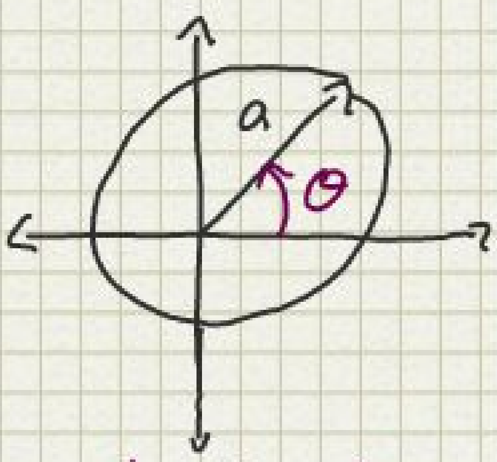
12.3 Integration in Polar Coordinates



$x = R\cos(\theta)$

$y = R\sin(\theta)$

$$\begin{cases} R^2 = x^2 + y^2 \\ \tan(\theta) = y/x \end{cases}$$

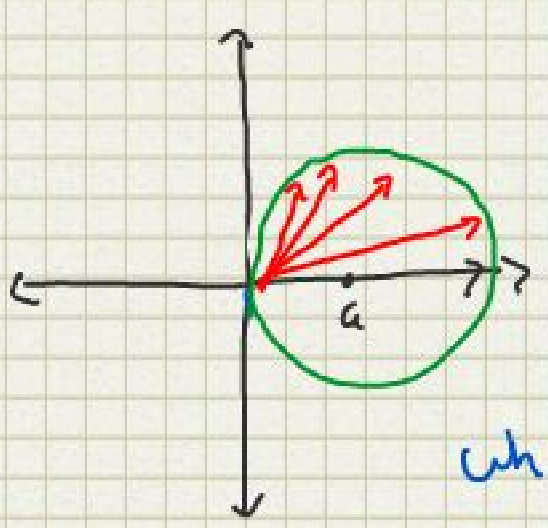


$R = a$ for all θ $\quad 0 \le \theta \le 2\pi$

$x^2 + y^2 = a^2$

$r^2 = a^2$

periodic in θ
in 2π



$(x-a)^2 + y^2 = a^2$

$x^2 - 2xa + a^2 + y^2 = a^2$

$R^2 - 2aR\cos(\theta) = 0$

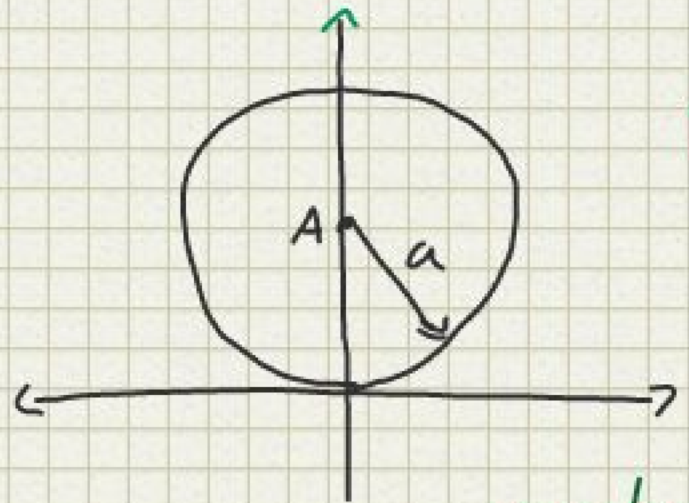
$R(R - 2a\cos(\theta)) = 0$ when $R = 0$

when $R \neq 0$

$(R - 2a\cos(\theta)) = 0 \quad R = 2a\cos(\theta)$

$0 \le \theta \le \pi$

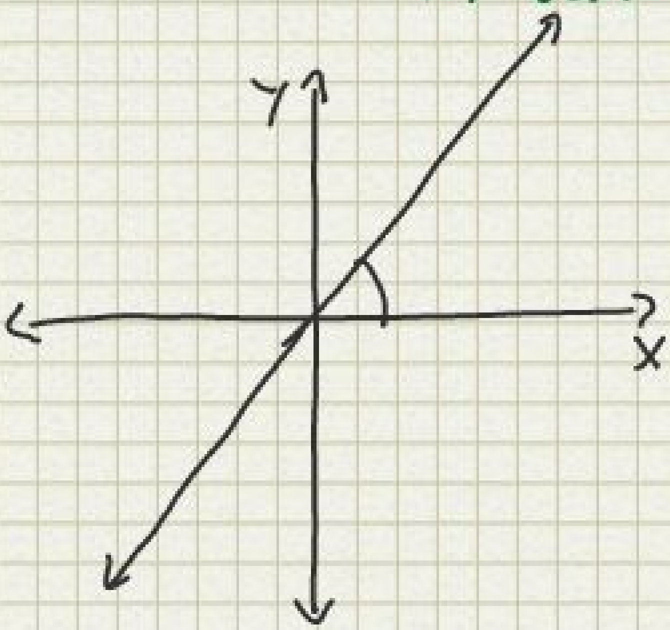
$-\pi/2 \le \theta \le \frac{\pi}{2}$



$r = 2a\sin(\theta) \quad 0 \leq \theta \leq \pi$

$m = \tan(\theta)$

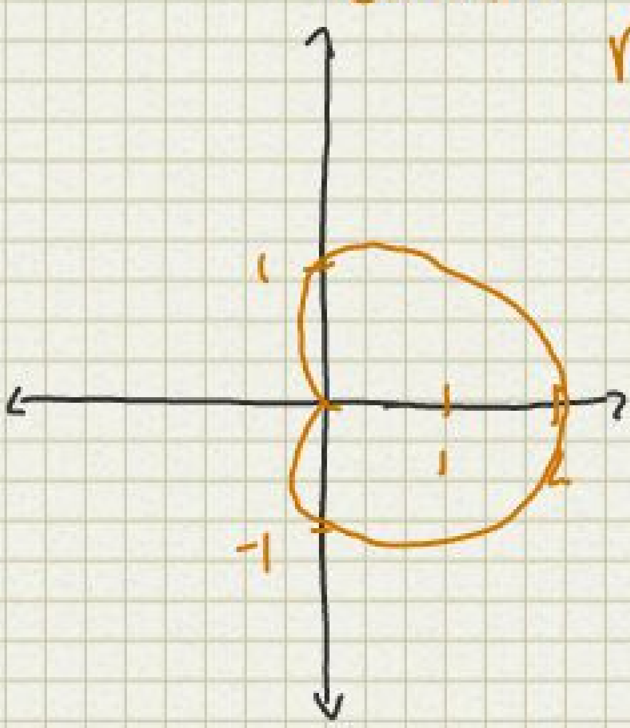
$\theta = \theta_0 \ \forall\, R \in (-\infty, \infty)$



Cardioc

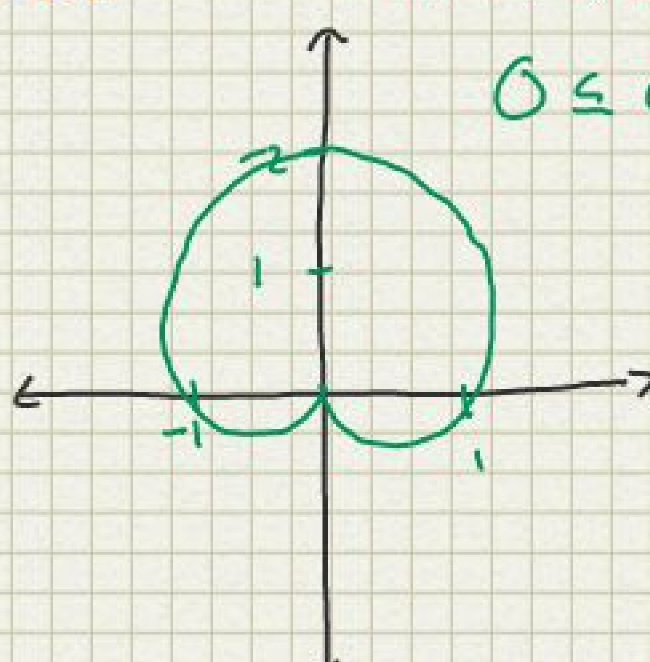
$r = 1 + \cos(\theta)$

$0 \leq \theta \leq 2\pi$



$R = 1 + \sin(\theta)$

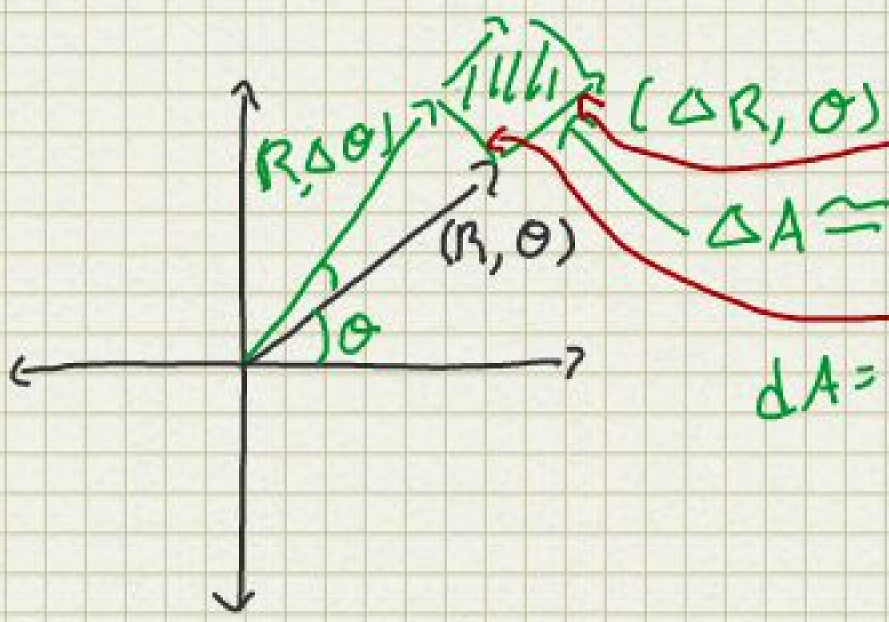
$0 \leq \theta \leq 2\pi$



$$\iint_D f(x,y)\,dA = \iint_D f(r\cos(\theta), r\sin(\theta))\,dA$$

dA: $\Delta x\,\Delta y$, $(\Delta x, \Delta y) \to (0,0)$

dA: $\Delta R, \Delta\theta$

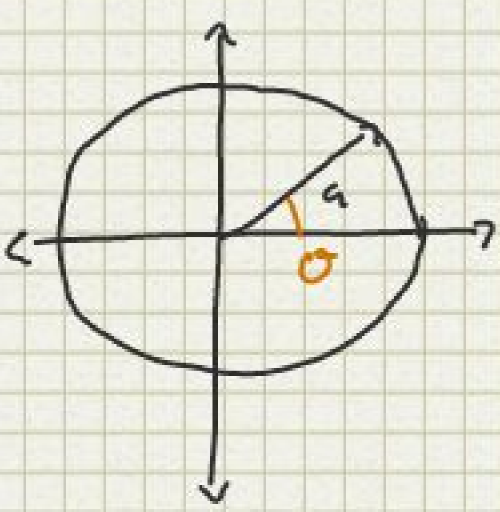


$$\Delta A \simeq R\Delta\theta \cdot \Delta R$$

$$dA = R\,dR\,d\theta$$

$$\int_{\theta_1}^{\theta_2} \int_{R_1(\theta)}^{R_2(\theta)} f(R,\theta)\,R\,dr\,d\theta$$

Find the area of a circle with radius "a"



$$A = \iint_D dA = \int_0^{2\pi} \int_{R=0}^{R=a} R\,dr\,d\theta$$

$$= \int_0^{2\pi} \frac{R^2}{2}\Big|_0^a d\theta = \int_0^{2\pi} \frac{a^2}{2}\,d\theta = \frac{a^2}{2}\theta\Big|_0^{2\pi}$$

$$= \frac{a^2}{2} 2\pi = \pi a^2$$