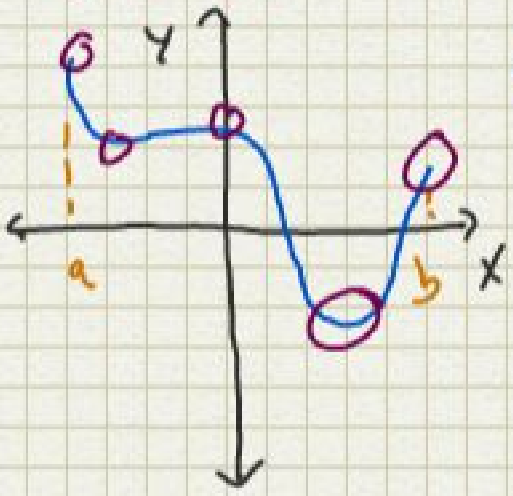
2/27/2014

11.7

Absolute Max or Min on D

(x_0, y_0) is a point absolute max for f in D
if for all x & y in D $f(x_0,y_0) \geq f(x,y)$
min $f(x_0,y_0) \leq f(x,y)$

If I is a closed interval $I=[a,b]$ and f is a smooth function in I then f has an absolute max and absolute min in I and they are obtained either in critical points inside I or on the boundary a or b.



max and mins have $\frac{d}{dx} = \emptyset$

Critical points inside D

C.P. $\begin{cases} f_x = \emptyset \\ f_y = \emptyset \end{cases}$

∂D is $\vec{R}(t) = \langle x(t), y(t) \rangle$

1st find all CP in D

2nd find all CP on boundary

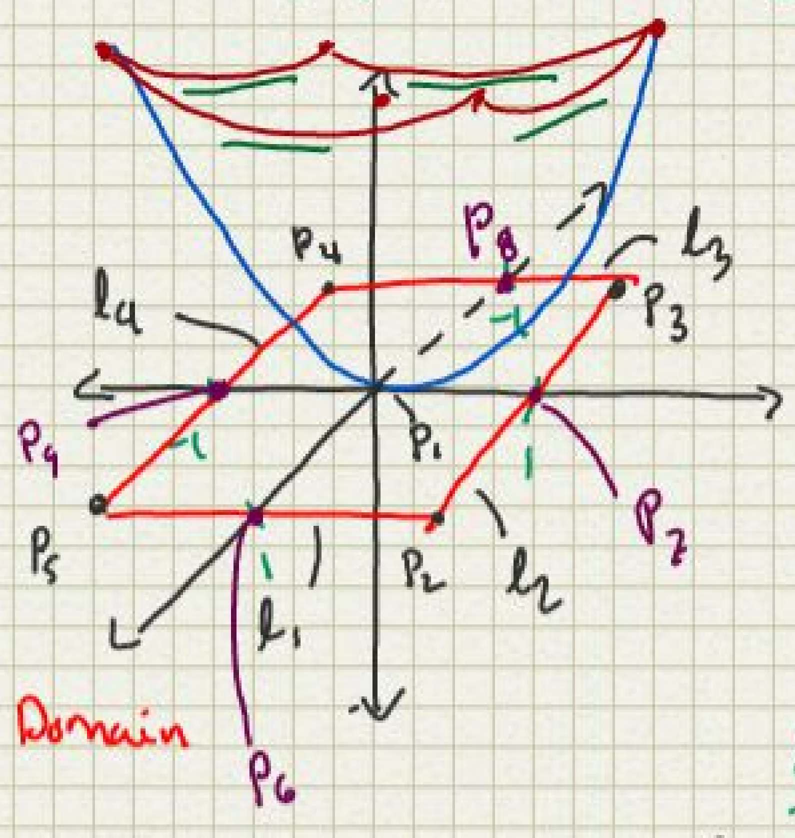
3rd find CP at edges

$g(t) = f(x(t), y(t)) \quad t_0 \leq t \leq t_1$

$g'(t) = \emptyset$

$g(t_0) = g(t_1) = ? \Rightarrow$ C.P.

Abs Max & Min $z = x^2 + y^2$ $D: \begin{cases} -1 \le x \le 1 \\ -1 \le y \le 1 \end{cases}$



Inside D: $\begin{cases} -1 < x < 1 \\ -1 < y < 1 \end{cases}$

$\begin{cases} f_x = 0 = 2x \\ f_y = 0 = 2y \end{cases} \Rightarrow P_1(0, 0)$

Square

l_1 $x = 1$ $-1 \le y \le 1$

l_2 $y = 1$ $-1 \le x \le 1$

l_3 $x = -1$ $-1 \le y \le 1$

l_4 $y = -1$ $-1 \le x \le 1$

$l_3 = l_1$ $g_1(y) = z(1, y) = 1 + y^2$

$g_1'(y) = 0 = 2y = 0$

$l_4 = l_2$ $g_2(x) = z(x, 1) = x^2 + 1$

$g_2'(x) = 0 = 2x$ $x = 0$

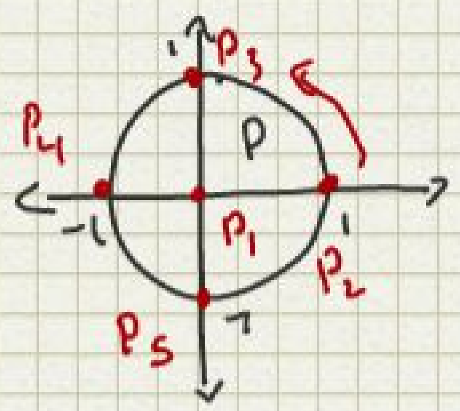
$f(P_1) = 0$

$f(P_i) = 2$ i $2 \to 5$

$f(P_i) = 1$ i $6 \to 9$

ABS Max & Min $\quad Z = e^{x^2-y^2}$

Inside $x^2+y^2 \le 1$



$x^2+y^2<1$

$$\begin{cases} Z_x = 0 = e^{x^2+y^2}(2x) \\ Z_y = 0 = e^{x^2+y^2}(-2y) \end{cases} \Longrightarrow \begin{cases} 2x = 0 \\ -2y = 0 \end{cases} P_1(0,0)$$

$x^2+y^2=1 \quad \begin{cases} y = \sqrt{1-x^2} \\ -1 \le x \le 1 \end{cases} \cup \begin{cases} y = -\sqrt{1-x^2} \\ -1 \le x \le 1 \end{cases}$

* $2\sin(t)\cos(t) = \sin(2t)$
* $\cos^2(t)-\sin^2(t) = \cos(2t)$

$\begin{cases} x(t) = \cos(t) \\ y(t) = \sin(t) \end{cases} \quad 0 \le t \le 2\pi \qquad P_2(1,0)$

$g(t) = Z(\cos(t), \sin(t)) = e^{\cos^2(t)-\sin^2(t)}$

$= e^{\cos(2t)}$

$g'(t) = e^{\cos(2t)}(-2\sin(2t)) = 0$

$0 < t < 2\pi$

$\sin(2t) = 0 \longrightarrow 2t = \pi, \ t = \pi/2 \quad (1)$

$\longrightarrow 2t = 2\pi \quad t = \pi \quad (2)$

$\longrightarrow 2t = 3\pi \quad t = \frac{3}{2}\pi$

$f(P_1) = e^0 = 1$

$f(P_2) = f(P_4) = e^1 = e \Longrightarrow$ Abs Max

$f(P_3) = f(P_5) = e^{0-1} = e^{-1} \Longrightarrow$ Abs min

11.8

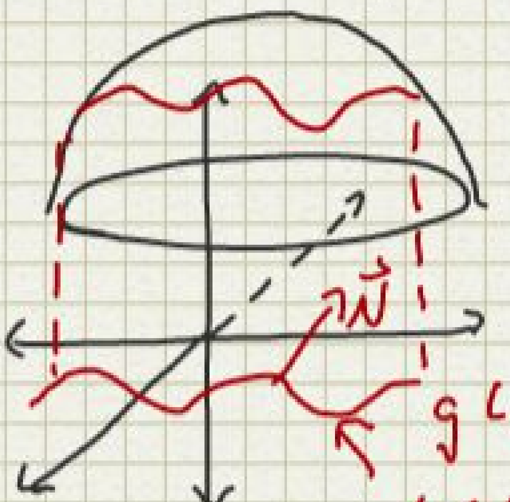
$f(x,y)$ is differentiable, has extrema on the curve $g(x,y)=c$ at the point (x_0,y_0)

then $\nabla f(x_0,y_0) = \lambda\, g(x_0,y_0)$ (line) (explicit)

(surface)
(implicit)

$g(x)$ is a curve on the plane

$g(x,y) = \vec{N}$



$g(t) = f(x(t), y(t))$

$g'(t) = 0 = \frac{d}{dt} f(x(t), y(t))$

$= f_x\, x'(t) + f_y\, y'(t) = \nabla f \cdot \vec{R}'(t) = 0$

$\nabla f \perp R'(t) \parallel \vec{T}(t) \Rightarrow \nabla f \parallel \vec{N} = \nabla g$

$\nabla f = \nabla g$

$$\boxed{\nabla f = \lambda \nabla g\,(x_0,y_0)}$$

$z = e^{x^2-y^2}$

D: $x^2+y^2=1$

$\nabla f = \langle 2x e^{x^2-y^2}, -2y e^{x^2-y^2} \rangle$

$g: x^2+y^2=1$ $\quad \nabla g = \langle 2x, 2y\rangle$ $\quad$ cp are points where $\nabla f = \nabla g$

$\longrightarrow$ always (+)

$$\begin{cases} 2x\, e^{x^2-y^2} = \lambda 2x \\ -2y\, e^{x^2-y^2} = \lambda 2y \\ x^2+y^2=1 \end{cases} \Longrightarrow \begin{cases} x=0 \\ (y=\pm 1) \\ y=1 \\ -2e^{-1} = \lambda 2 \end{cases} \quad \lambda = -e^{-1} \Rightarrow P_3 + P_5$$

$$\begin{cases} y=0 \\ (x=\pm 1) \\ 2(\pm 1)\, e^{1-0} = \lambda 2(\pm 1) \end{cases}$$

Assume $f(x,y,z) = x^2 + y^2 + z^2$

abs max $\underline{x - 2y + 3z = 4}$ Restricting search for abs max on plane

$$\begin{cases} \nabla f = \lambda \nabla g \\ g(x,y,z) = c \end{cases} \qquad g: x - 2y + 3z = 4$$

$$\begin{cases} 2x = \lambda(1) \\ 2y = \lambda(-2) \\ 2z = \lambda(3) \\ x - 2y + 3z = 4 \end{cases} \qquad \begin{cases} x = \frac{\lambda}{2} \\ y = -\lambda \\ z = \frac{3}{2}\lambda \\ \frac{\lambda}{2} + 2\lambda + \frac{9}{2}\lambda = 4 \end{cases} \qquad \begin{cases} x = \frac{4}{14} \\ y = -\frac{4}{7} \\ z = \frac{12}{14} \\ \lambda = \frac{4}{7} \end{cases}$$

$$7\lambda = 4$$

$$\lambda = \frac{4}{7}$$