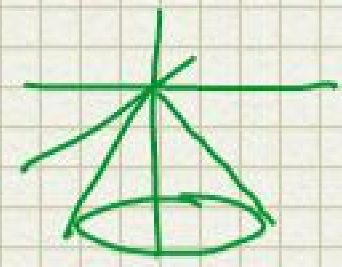
2/26/2014

Relative Min

Relative Max



Saddle point

Smooth Critical points $\begin{cases} f_x(x_c, y_c) = 0 \\ f_y(x_c, y_c) = 0 \end{cases} \Longrightarrow$ Horizontal tangent Plane

Find

$z = f(x_c, y_c)$

horizontal and passes through x_c, y_c

2nd Order Partial Deriv. Test

$$Df(x_c,y_c) = f_{xx}(x_c,y_c)\, f_{yy}(x_c,y_c) - f_{xy}^2(x_c,y_c)$$

D:
- $< 0 \Rightarrow$ saddle point
- $0 \Rightarrow$ Test is inconclusive
- f_{xx} or $f_{yy} > 0$ Rel Min
- f_{xx} or $f_{yy} < 0$ Rel Max

Classify

<u>Find and Classify all critical points</u>

$f(x,y) = 2x^2 + 2xy + y^2 - 2x - 2y + 5$

$f_x = 4x + 2y - 2 \quad f_y = 2x + 2y - 2$

$$\begin{cases} f_x = 4x + 2y - 2 = 0 \\ f_y = 2x + 2y - 2 = 0 \end{cases}$$

$2x = 0 \quad x = 0$

(Plug x-value into f_y)

$2(0) + 2y - 2 = 0 \quad y = 1$

$\Rightarrow P(0,1)$

$f_{xx} = 4$

$f_{yy} = 2$

$f_{xy} = 2$

$Df(0,1) = 4(2) - 2^2 = 4 > 0$

$f_{xx}(0,1) = 4 > 0 \Rightarrow$ Rel min

$f(x,y) = 8x^3 - 24xy + y^3$

Find and Classify all critical points

$f_x = 24x^2 - 24y \quad f_x = -24x + 3y^2$

$$\begin{cases} f_x = \emptyset = 24x^2 - 24y & (1) \\ f_y = \emptyset = 3y^2 - 24x & (2) \end{cases} \quad \begin{array}{l} f_{xx} = 48x \quad f_{xy} = -24 \\ f_{yy} = 6y \end{array}$$

(1) $y = x^2$ (2) $3(x^2)^2 - 24x = \emptyset$

$3x(-8 + x^3) = \emptyset \Rightarrow \begin{cases} x = \emptyset \\ y = \emptyset^2 \end{cases}$ $P_1(\emptyset, \emptyset)$

$\Downarrow$

$\begin{cases} x^3 = 8 \\ y = x^2 \end{cases} = \begin{cases} x = 2 \\ y = 4 \end{cases}$

$P_2(2,4)$

$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = f_{xx} f_{yy} - f_{xy}^2$

$D = (48x)(6y) - (-24)^2$

$= (24 \cdot 12)(xy - 2)$

$Df(P_1) = (24 \cdot 12)(\emptyset - 2) < \emptyset$ (saddle point)

$Df(P_2) = (24 \cdot 12)(8 - 2) > \emptyset \Rightarrow f_{xx}(P_2) = 48 > \emptyset$

$\Rightarrow P_2$ is Relative Min

$f(x,y) = x^2 y^4$

∞ solutions

$$\begin{cases} f_x = 0 = 2xy^4 \\ f_y = 0 = 4x^2y^3 \end{cases}$$

$x = 0 \Rightarrow \begin{cases} f_x = 0 \\ f_y = 0 \end{cases}$ for all y

$y = 0 \Rightarrow \begin{cases} f_x = 0 \\ f_y = 0 \end{cases}$ for all x

$D f = f_x f_y - f_{xy}^2$

$= (2y^4)(x^2 12y^2) - (2x\,4y^3)^2$

$24x^2y^6 - 64x^2y^6 = -40x^2y^6$

$Df(x=0) = 0$

$Df(y=0) = 0$ Test Fails

$f(x,y) = x^2y^4 \geq 0$ $\quad f(x=0) = f(y=0) = 0$

function consists of squares so the function and its derivatives will always be positive $\Rightarrow$ the function will have a relative minimum