2/19/2014

$$\frac{d}{dx}\left(\ln(xy + yz + z) = 5\right)$$

z_x z_y

Intersted in solving z_x

$z(x,y)$

$$\frac{1}{xy + yz + xz}\left(y + yz_x + z + xz_x\right) = \emptyset$$

$$\frac{A(x)}{B(x)} = \emptyset \quad A(x) = \emptyset \quad B(x) \neq \emptyset \quad x, y > \emptyset$$

$A(x) = \emptyset$

$y + yz_x + z + xz_x = \emptyset$

$(y + x)z_x = -x - y \quad z_x = \dfrac{-z - y}{y + x}$

$$\frac{1}{xy + yz + xz}\left(y + y3z^3 z_x + z + x(z_x)\right) = \emptyset$$

$$z_x = \frac{-z - y}{3z^2 y + x}$$

$y + 3yz^2 z_x + z + xz_x = \emptyset$

$3yz^2$

$w = \sqrt{x + 2y + 3z}$ $\quad$ w is a number

$w^2 = x + 2y + 3z$ $\quad$ $w = 1 \Rightarrow 1$

(plane) $\quad$ $w = 2 \Rightarrow 4$

$w = 3 \Rightarrow 9$

unequally spaced

$w = \sqrt{x^2 + y^2 + z^2}$

$w^2 = x^2 + y^2 + z^2$ $\quad$ $w^2 = r^2$

(sphere) equally spaced

$w = x + 2y + 3z$

planes equally spaced

$w = \sqrt{x^2 + 2y^2 + 3z^2}$

$w^2 = x^2 + 4y^2 + 9z^2$

$1 = \frac{x^2}{w^2} + \frac{y^2}{\left(\frac{w}{2}\right)^2} + \frac{z^2}{\left(\frac{z}{3}\right)^2}$ $\quad$ equilly spaced ellipsoid

$w = x^2 + 2y^2 + 3z^2$ $\quad$ concentric ellipsoid

$\frac{x^2}{\sqrt{w}} + \frac{y^2}{\left(\frac{\sqrt{w}}{2}\right)^2} + \frac{z^2}{\left(\frac{\sqrt{w}}{3}\right)^2}$

$w = x^2 - y^2 - z^2$

cones

$1 = \frac{x^2}{(\sqrt{w})^2} - \frac{y^2}{(\sqrt{w})^2} - \frac{z^2}{(\sqrt{w})^2}$ two sheet hyperballoid

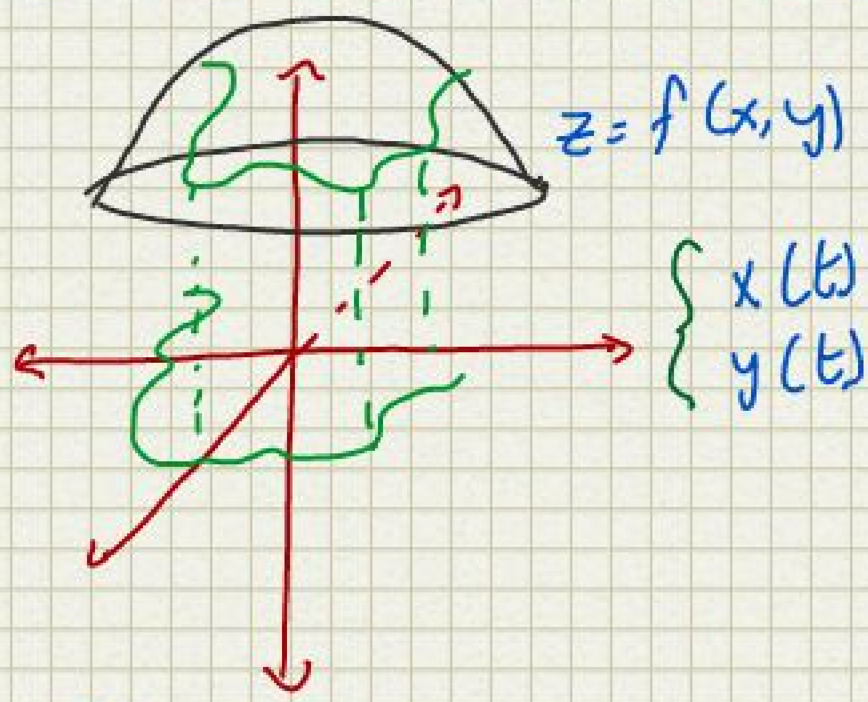
$w > \emptyset$: two sheet

$w = \emptyset$: cone

$w < \emptyset$: one sheet

2/20/2014

Chain rule in several variables



$$\begin{cases} x(t) \\ y(t) \end{cases} \quad z(t) = f(x(t), y(t))$$

$$z'(t) = \frac{dz}{dt}$$

Chain Rule

$\frac{dz}{dt}$ is the

$$\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

$$\frac{df}{dt} = f_x \frac{dx}{dt} + f_y \frac{dy}{dt}$$

Ex

$$z = x^2 + y^2 \qquad \left(x = \frac{1}{t}, y = t^2\right)$$

Substitution

$$z(t) = \left(\frac{1}{t}\right)^2 + (t^2)^2$$

$$= \frac{1}{t^2} + t^4$$

$$\frac{dz}{dt} = -2t^{-3} + 4t^3$$

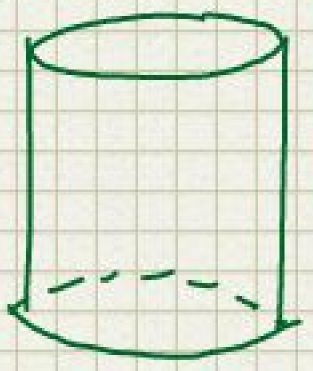
Chain Rule

$$z_t = f_x \frac{dx}{dt} + f_y \frac{dy}{dt} = 2x\left(\frac{-1}{t^2}\right) + 2y(2t)$$

$$= 2\left(\frac{1}{t}\right)\left(-\frac{1}{t^2}\right) + 2t^2 2t$$

$$= \frac{-2}{t^3} + 4t^3$$

Chain Rule: $z_t = f_x \frac{dx}{dt} + f_y \frac{dy}{dt}$



$\frac{dr}{dt} = 3 \frac{in}{min}$

$\frac{dh}{dt} = -5 \frac{in}{min}$

$R = 10\,in \quad h = 8\,in$

$V(r,h) = \pi r^2 h$

$\frac{dV}{dt}(10,8) = ?$

$$\frac{dV}{dt} = V_r \frac{dr}{dt} + V_h \frac{dh}{dt}$$

$$= 2\pi r h(3) + \pi r^2(-5)$$

$$\frac{dV}{dt}(10,8) = 2\pi(10)(8)(3) + (-5)(\pi)(10^2)$$

$$= 480\pi - 500\pi = -20\pi \frac{in^3}{min}$$

$F(x,y,z) = c$

$z_x \quad z_y$

$x(t) = t$

$y(t) = 0$

Apply chain rule

$$F_x \frac{dx}{dt} + F_y \frac{dy}{dt} + F_z \frac{dz}{dt} = 0$$

$t = x$

$$F_x \underbrace{\cancel{\frac{dx}{dx}}}_{1} + F_y \underbrace{\cancel{\frac{dy}{dx}}}_{0} + F_z \frac{dz}{dx} = 0$$

$$z_x = \frac{-F_x}{F_z} \qquad z_y = \frac{-F_y}{F_z}$$

$= \ln(xy + xz + yz) = 5 \qquad x, y > 0$

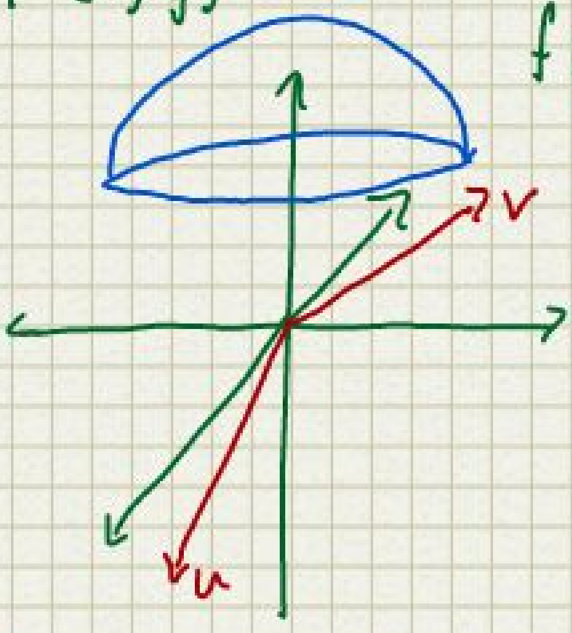
$F(x,y,z)$

z_x, z_y

$$f_x = \frac{y+z}{xy+yz+xz} \qquad F_y = \frac{x+z}{xy+yz+xz} \qquad f_z = \frac{x+y}{xy+yz+xz}$$

$$z_x = -\left(\frac{y+z}{x+y}\right) \qquad z_y = -\left(\frac{x+z}{x+y}\right)$$

$F(x,y)$



$f(u,v)$

$x(u,v)$

$y(u,v)$

$$\frac{\partial f}{\partial u} \quad \frac{\partial f}{\partial v}$$

$$\frac{\partial f}{\partial u} = f_x(x,y)\frac{\partial x}{\partial u} + f_y(x,y)\frac{\partial y}{\partial u}$$

$$\frac{\partial f}{\partial v} = f_x(x,y)\frac{\partial x}{\partial v} + f_y(x,y)\frac{\partial y}{\partial v}$$

$f(x,y) = e^{x^2 y}$ $\quad \frac{\partial f}{\partial u}, \frac{\partial f}{\partial v}$

use: $x = u+v$

$y = u-v$

$f_x = e^{x^2 y}(2xy)$

$f_y = e^{x^2}(x^2)$

$x_u = 1 \quad x_v = 1$

$y_u = 1 \quad y_v = -1$

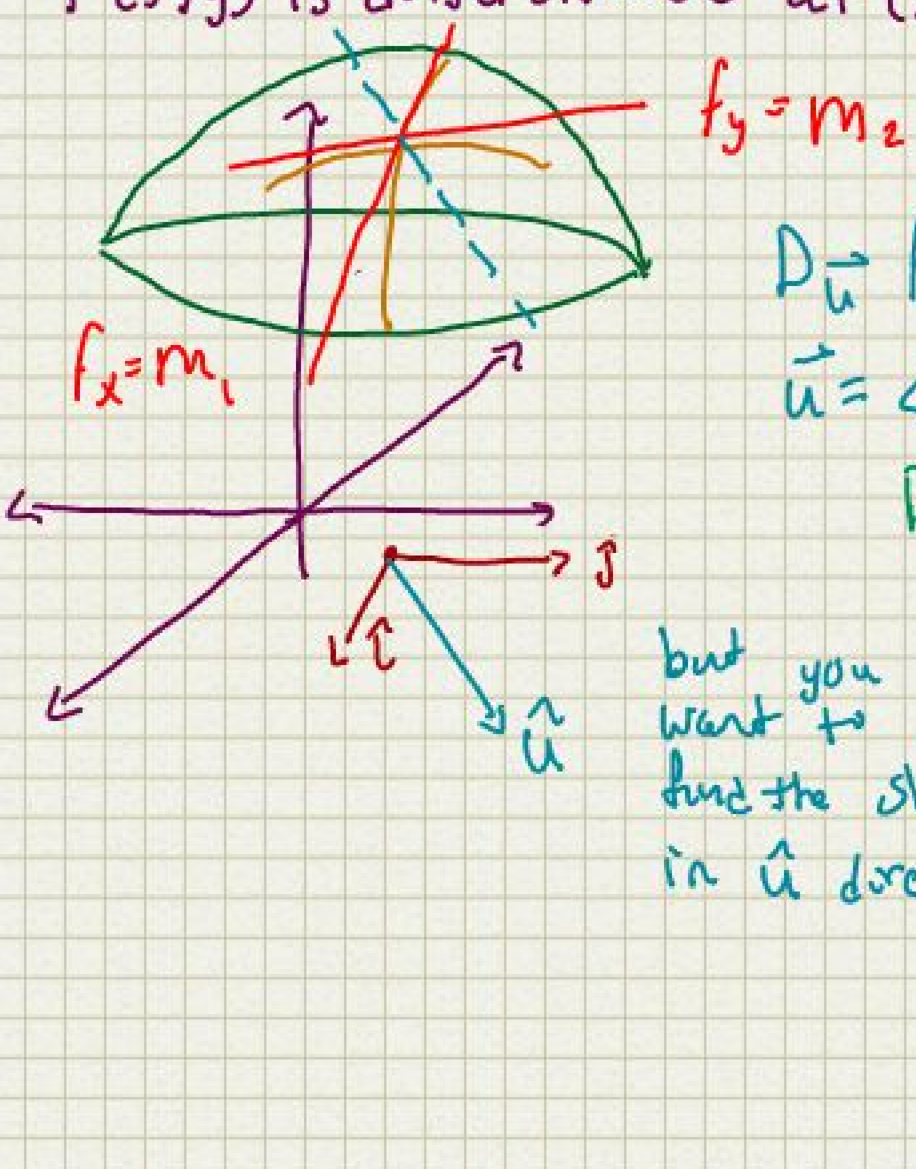
$f_u = 2xy\, e^{x^2 y}(1) + e^{x^2 y} x^2 (1)$

$f_v = 2xy\, e^{x^2 y}(1) + e^{x^2 y} x^2 (-1)$

11.6 Directional Derivative

Consider the condition

$f(x,y)$ is differentiable at (x_0, y_0)



$D_{\vec{u}} f(x,y) = f_x(x_0,y_0)u_1 + f_y(x_0,y_0)u_2$

$\vec{u} = \langle u_1, u_2 \rangle = u_1 \hat{i} + u_2 \hat{j}$

$D_{\hat{i}} f = f_x(x_0,y_0)(1) + \emptyset$

$D_{-\hat{i}} f = -f_x(x_0,y_0)(1) + \emptyset$

but you want to find the slope in $\hat{u}$ direction

$D_{\hat{u}} f(x,y) = -D_{-\hat{u}} f(x,y)$

Proving $\vec{u} = 1$

$D_{\vec{u}} f(x,y)$ $\vec{u} \parallel \vec{v}$ $\langle 3,4 \rangle$ $\quad \vec{u} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{\langle 3,4 \rangle}{\sqrt{3^2+4^2}} = \frac{\langle 3,4 \rangle}{5}$

$f(x,y) = \cos(x+y)$

$(x_0, y_0) = (\pi/4, \pi/4)$

$f_x = -\sin(x+y)(1) \quad f_x(x_0,y_0) = -\sin\left(\frac{\pi}{4}+\frac{\pi}{4}\right) = -1$

$f_y = -\sin(x+y)(1) \quad f_y(x_0,y_0) = -1$

$D_{\vec{u}} f\left(\frac{\pi}{4}, \frac{\pi}{4}\right) = -1\left(\frac{3}{5}\right) - 1\left(\frac{4}{5}\right) = -\frac{7}{5}$

first solve unit vector parallel

take partial derivatives at (x_0, y_0)

Gradient Operator

∇f, ∇ del: $\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \rangle$

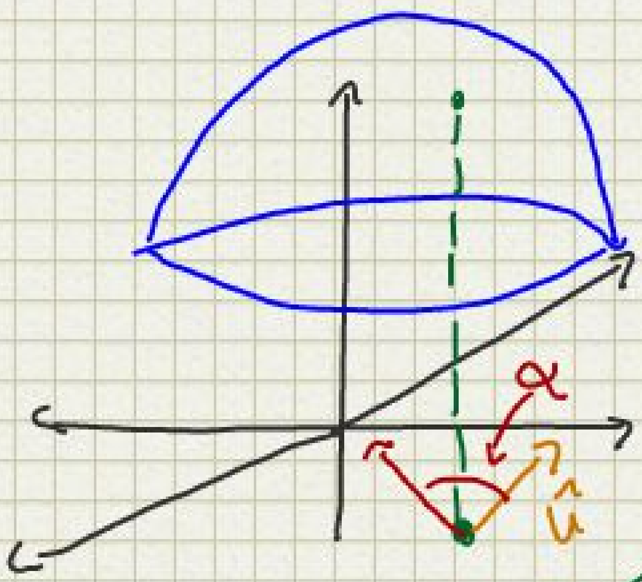
$\nabla f = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \rangle f$

$= \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \rangle$

$D_{\vec{u}} f = f_x u_1 + f_y u_2 = \nabla f \cdot \vec{u} = \langle f_x, f_y \rangle \cdot \langle u_1, u_2 \rangle$

$\nabla f \cdot \vec{u} = \|\nabla f\| \, \|\vec{u}\| \cos(\alpha)$ α is the angle between ∇f & $\hat{u}$

$= \|\nabla f\| \cos(\alpha)$ $\|\hat{u}\|$ disappeared b/c its on x,y axis



$\nabla f(x_0, y_0) = \langle f_x(x_0, y_0), f_y(x_0, y_0)\rangle$

$= (3, 4)$

$D_{\hat{u}} f(x_0, y_0) = \|\nabla f(x_0, y_0)\| \cos(\theta)$

greatest increase → when $\alpha = \emptyset \Rightarrow D_{\vec{u}} f(x_0, y_0) = \|\nabla f(x_0, y_0)\|$

decrease → when $\alpha = -180 = D_{\vec{u}} f(x_0, y_0) = -\|\nabla f(x_0, y_0)\|$

no increase/decreas → when $\alpha = \frac{\pi}{2} \Rightarrow D_{\vec{u}} f(x_0, y_0) = \emptyset$

Ex

$f(x,y) = 3x^2 + xy + y$ Find the direction at (x_0, y_0) for which you have max rate of increase + find rate

$(x_0, y_0) = (1, -1)$

$\vec{V} = \nabla f(x_0, y_0)$

max rate $\|\nabla f(x_0, y_0)\|$

$\nabla f = \langle \underbrace{6x+y}_{f_x}, \underbrace{1+x}_{f_y} \rangle$ $\quad \vec{v} = \nabla f(1, -1) = (6-1, 1+1)$

$= \langle 5, 2 \rangle$

max rate $= \|\nabla f(x_0, y_0)\| = \sqrt{5^2 + 2^2} = \sqrt{29}$