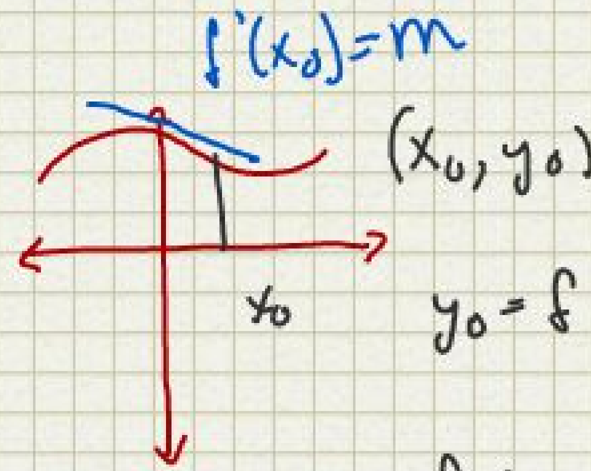
2/18/2014

11.4

# Tangent Planes

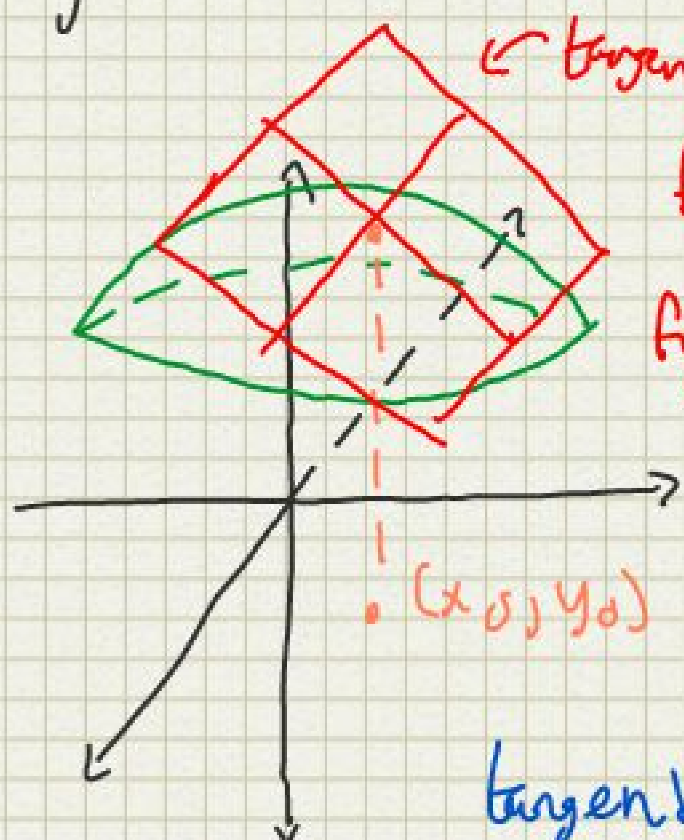
From calc I




$y_0 = f(x_0)$

$y - y_0 = m(x - x_0)$

$y - f(x) = f'(x)(x - x_0) \longrightarrow y = f(x_0) + f'(x_0)(x - x_0)$




$f_x(x_0, y_0) = m_1$

$f_y(x_0, y_0) = m_2$

tangent plane

$$\begin{cases} z - f(x_0,y_0) = f_x(x_0,y_0)(x - x_0) + f_y(x_0,y_0)(y - y_0) \\ y = y_0 \end{cases}$$

intersecting plane on y-axis

$$\begin{cases} y = y_0 \\ z - f(x_0 y_0) = f_x(x_0, y_0)(x - x_0) \end{cases}$$

$z - z_0 = m_1(x - x_0)$

intersecting plane on x-axis

$$\begin{cases} x = x_0 \\ z - z_0 = m_2(y - y_0) \end{cases}$$

Ex: $f(x,y) = \cos(x+y)$

Tangent Plane? at $(x_0, y_0) = (\pi/4, 0)$

$f(x_0,y_0) = \cos(\pi/4 + 0) = \frac{\sqrt{2}}{2} = z_0$

$f_x(x_0,y_0) = -\sin(x+y)(1)$

$f_y(x_0,y_0) = -\sin(x+y)(1)$

$f_x(\pi/4, 0) = -\sin(\pi/4 + 0) = -\frac{\sqrt{2}}{2}$

$f_y(\pi/4, 0) = -\frac{\sqrt{2}}{2}$

$(z - \frac{\sqrt{2}}{2}) = -\frac{\sqrt{2}}{2}(x - \pi/4) + (-\frac{\sqrt{2}}{2})(y - 0)$

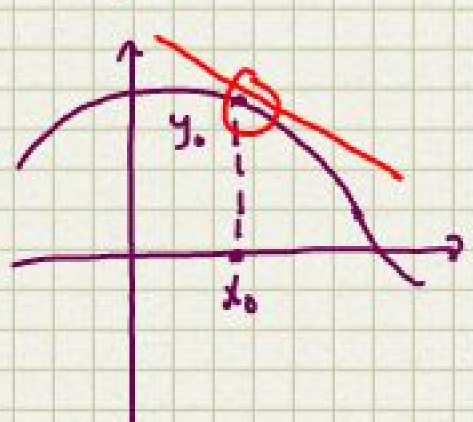
* Remember to evaluate $z_0$, $f_x$, $f_y$ + plug into equation of Plane

$z + \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y = \frac{\sqrt{2}}{2} + \frac{\pi\sqrt{2}}{8}$

$\vec{N} = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1)$

Incremental Approximation

- Tangent line best approx curves with proximity of point

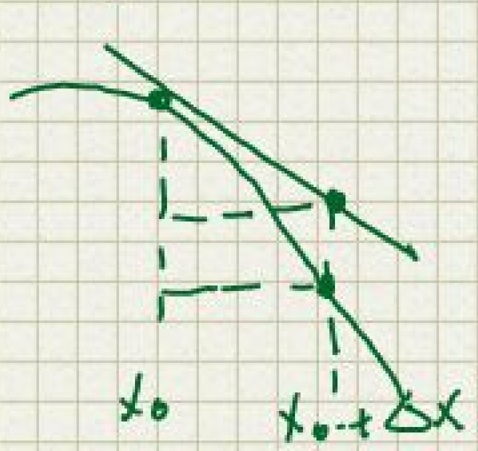



$y - y_0 = f'(x_0)(x - x_0)$ (equation of line)

$f - y_0 \simeq f'(x_0)(x - x_0)$ (approximation of function)

$f - f(x_0) \simeq f'(x_0)(x - x_0)$

$\Delta f \simeq f'(x_0)\Delta x$



# Incremental Approximation

$$f(x,y)-f(x_0,y_0)\simeq f_x(x_0,y_0)(x-x_0)+f_y(x_0,y_0)(y-y_0)$$

$$\Delta f \simeq f_x(x_0,y_0)\Delta x + f_y(x_0,y_0)\Delta y$$

Inc Approx for function $f(x,y)$

## Open Box

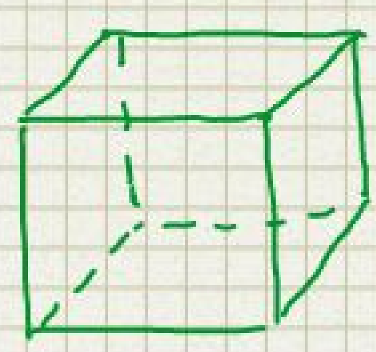
length 3 ft $\quad \Delta C$ if length and width increased 3 in

width 1 ft $\quad$ height - 4 in

height 2 ft

\$2/ft$^2$ for sides

\$3/ft$^2$ for bottom



$$C(l,w,h)=3lw+2(2wh+2lh)$$

$$C(3,1,2)=3\cdot 3\cdot 1+2(2\cdot 1\cdot 2+2\cdot 3\cdot 2)=\$41$$

$$\Delta C \simeq C_l(3,1,2)\underbrace{(0.25)}_{\Delta l}+C_w(3,1,2)\underbrace{(0.25)}_{\Delta w}+C_h(3,1,2)\underbrace{(-1/3)}_{\Delta h}$$

- $C_l = 3w+4(h)$

$C_l(3,1,2)=3(1)+4(2)=11$

- $C_w = 3l+4h$

$C_w=(3,1,2)=3(3)+4(2)=17$

- $C_h = 4(w+l)$

$C_h=(3,1,2)=4\cdot 4=16$

$$\Delta C \simeq 11\cdot \frac{1}{4}+17\cdot\frac{1}{4}-\frac{16}{3}$$

$$=7-\frac{16}{3}=\$\frac{5}{3}$$

# Differentials

$\Delta f \simeq f'(x)\Delta x$

* differentials are infinitesimal numbers (zero)

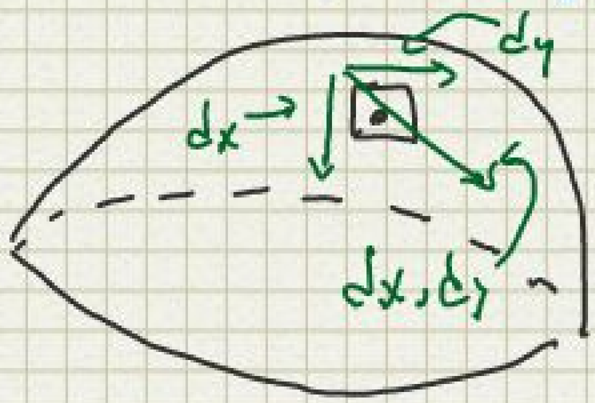
$\lim \Delta x \to 0$

$df = f'(x)\,dx$

$\Delta f \simeq f_x(x,y)\Delta x + f_y(x,y)\Delta y$

$\Delta x \to 0,\ \Delta y \to 0$

$df = f_x(x,y)\,dx + f_y(x,y)\,dy$




$f(x,y) = e^{xy}y^2$

$$df = \underbrace{e^{xy}(y)y^2\,dx}_{f_x} + \underbrace{\left(e^{xy}(x)(y^2) + e^{xy}(2y)\right)}_{f_y} dy$$

$df(1,2) = e^2(2)^3 + e^2(1)2^2 + e^2(4)$

$= 8e^2\,dx + 8e^2\,dy$

Differentials

Partial Derivatives w/ respect to x + y

Differentiation of function at $(x,y)$

- function is differentiable if there exists a tangent plane

A function $f$ is differentiable at $(x_0, y_0)$ if and only if

$f$ is continuous at $(x_0, y_0)$

$f_x(x_0, y_0)$ exists and is continuous

$f_y(x_0, y_0)$ exists and is continuous

* all three arguements have to be true for a function to be differentiable