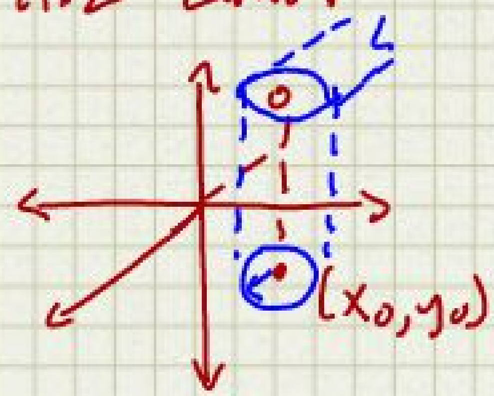
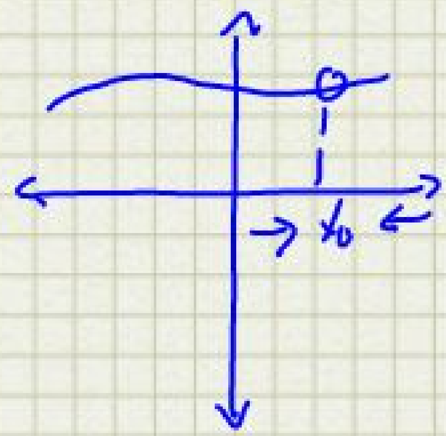
11.2 Limit



$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$$



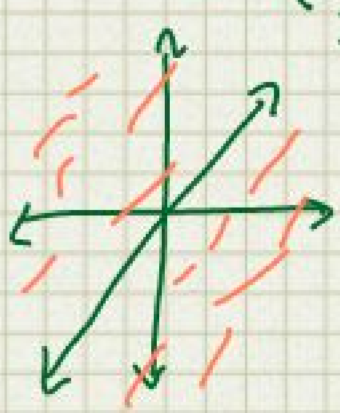
$$\lim_{x\to x_0^-} f(x) = L = \lim_{x\to x_0^+} f(x)$$

$$\lim_{x\to x_0} f(x) = L$$

$$\lim_{(x,y)\to(\emptyset,\emptyset)} \frac{(x-y)(x+1)}{(x-y)} = \frac{\emptyset}{\emptyset}$$ limit is undetermined

$\frac{\emptyset}{\emptyset}$ $\left.\begin{matrix}\emptyset \\ a\end{matrix}\right\}$ defined $\quad (x-y) \neq \emptyset$

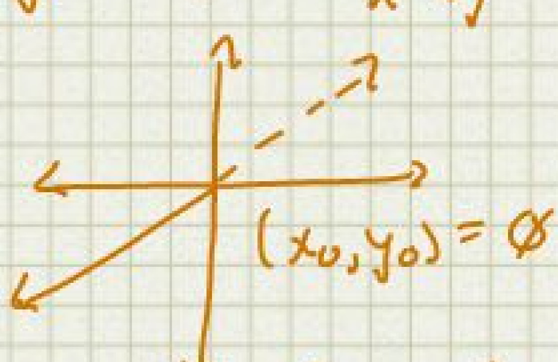
$\pm\infty \to$ undefined



$$\lim_{(x,y)\to(\emptyset,\emptyset)} \frac{\cancel{(x-y)}(x+1)}{\cancel{(x-y)}} = 1$$

$$\lim_{(x,y)\to(\emptyset,\emptyset)} \frac{2xy}{x^2+y^2} = \frac{\emptyset}{\emptyset}$$ DNE

*different values

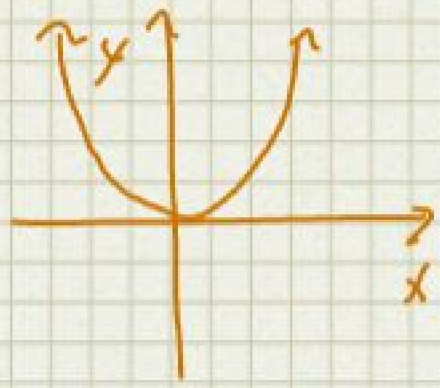


$$\lim_{(x,mx)\to(\emptyset,\emptyset)} f(mx) = \lim_{x\to 0} \frac{2x\,mx}{x^2+mx^2} = \frac{2m}{1+m^2}$$

$$\lim_{(x,y)\to(\emptyset,\emptyset)} \frac{x^2y}{x^4+y^2} = \frac{\emptyset}{\emptyset} \quad y = mx$$

$$\lim_{x\to 0} f(x, mx) = \lim_{x\to\emptyset} \frac{x^2\,mx}{x^4+m^2x^2} = \frac{mx}{x^2+m^2} = \frac{\emptyset}{m^2} = \emptyset$$

$$y = x^2 \quad \lim_{x\to 0} f(x, x^2) = \lim_{x\to\emptyset} \frac{x^2x^2}{x^4+x^4} = \frac{1}{2} \quad \underline{DNE}$$



$$\lim_{(x,y)\to(\emptyset\emptyset)} f(x,y) \quad \begin{matrix} x\ r\cos(\theta) \\ y = r\sin(\theta) \end{matrix} \quad \lim_{r\to\emptyset} f(r\cos(\theta), r\sin(\theta)) = L(\theta) = L$$

$$\lim_{(x,y)\to(\emptyset,\emptyset)} \frac{2xy}{x^2+y^2} = \lim_{r\to\emptyset} \frac{2r\cos(\theta)\,r\sin(\theta)}{R^2\cos^2(\theta)+R^2\sin^2(\theta)}$$

$$= 2\cos(\theta)\sin(\theta) = \sin(2\theta)$$

$$= DNE$$

$$\lim_{(x,y)\to(\emptyset,\emptyset)} \frac{\tan(R^2)}{R^2}$$

$$= \lim \frac{\sin(p)}{p} = \lim_{p\to\emptyset} \frac{\sin(p)}{p}\,\frac{1}{\cos(p)} = 1$$

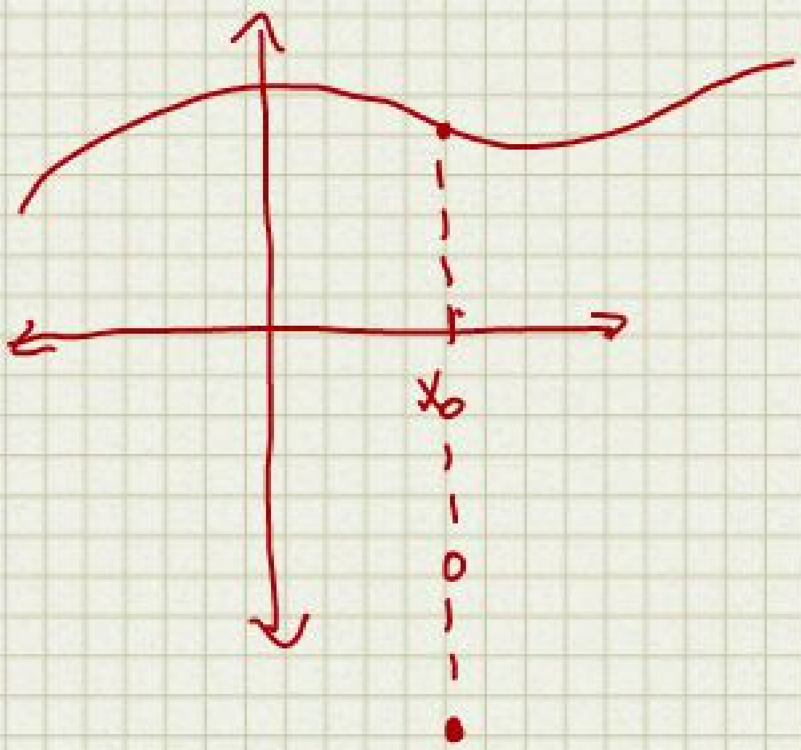
$$\lim_{t\to 0} \frac{\sin(t)}{t} \overset{L'H}{=} 1$$

Continuity

$f(x_0) =$ defined

$\lim_{x \to x_0} f(x) = L$

$f(x_0) = L$

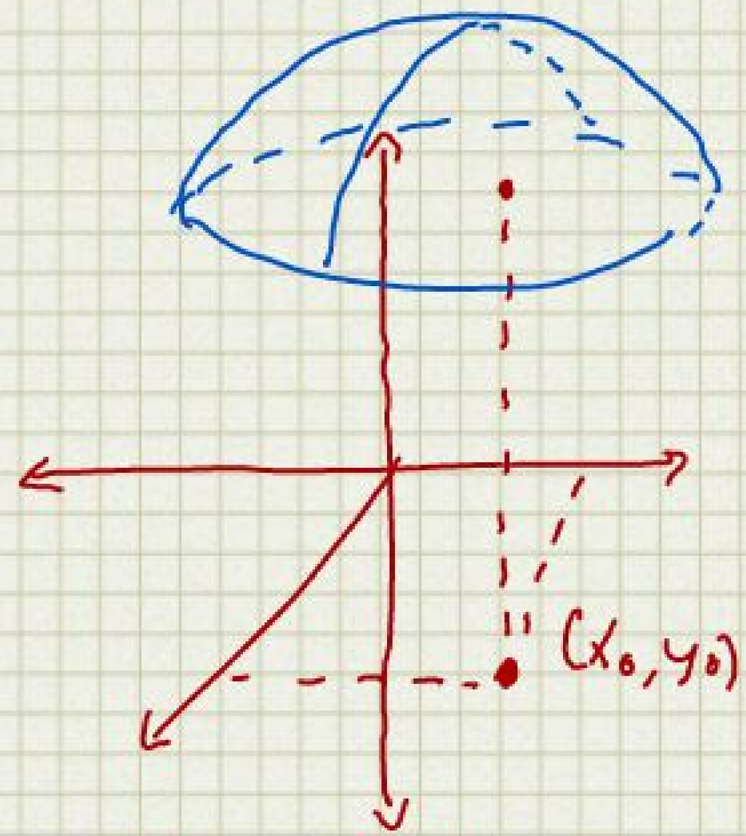


11.2 Limits

f is continuous at (x_0, y_0)

if $f(x_0, y_0)$ exists

$f(x_0, y_0) = L$



Standard Function

$p_n t$

$p_n t \quad q = \frac{p_n}{R_n}$ is continuous at $\sin(q)$ $\cos(q)$ $\tan(q)$

(is continuous)

$f(x) = \tan(x^2 + y^2)$

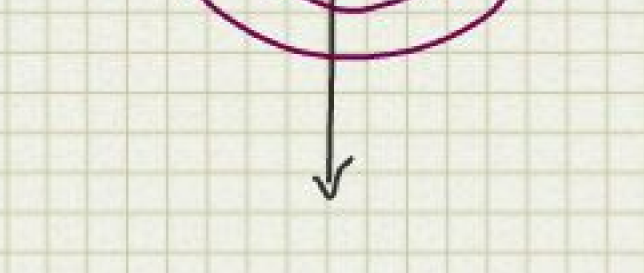
$x^2 + y^2 \neq \pi/2 + 2\pi$

$f = \ln(xy)$ ← inside always has to be positive

⟸ therefore

$xy > \emptyset$: → $(+)x_0, (+)y_0$

→ $(-)x_0, (-)y_0$



11.3 Partial Derivatives

- Function of several variables

$f(x,y) \quad f(x,y,z)$

- a function $f(x,y)$ has partial derivatives w/ respect to x (or y) at (x_0, y_0) if the limit:

for x $\rightarrow$ $\lim_{h \to 0} \frac{f(x_0+h, y_0) - f(x_0, y_0)}{h}$ exists

and in that case we use the notation $f_x(x_0, y_0)$ or $\frac{\partial f(x_0, y_0)}{\partial x}$

or $\left.\frac{\partial f}{\partial x}\right|_{(x_0, y_0)}$.

Partial Derivative with respect to y

$$f_y(x_0, y_0) = \frac{\partial f}{\partial y}(x_0, y_0) = \left.\frac{\partial f}{\partial y}\right|_{(x_0, y_0)}$$

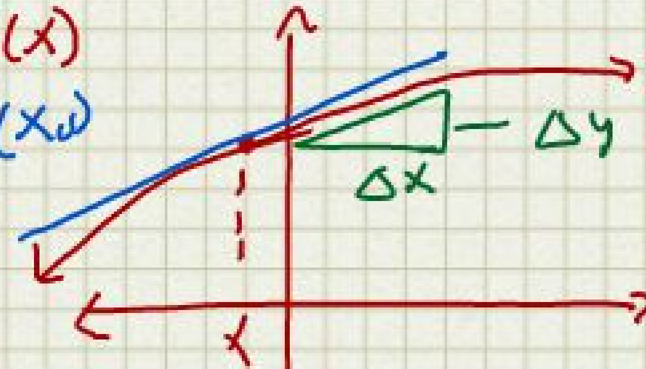
$$= \lim_{h \to 0} \frac{f(x_0, y_0+h) - f(x_0, y_0)}{h}$$

$$f_x(x,y) = \lim_{h \to 0} \frac{f(x+h, y) - f(x,y)}{h}$$

$$f_y(x,y) = \lim_{h \to 0} \frac{f(x, y+h) - f(x,y)}{h}$$

$f(x,y)$

$f(x,y,z)$



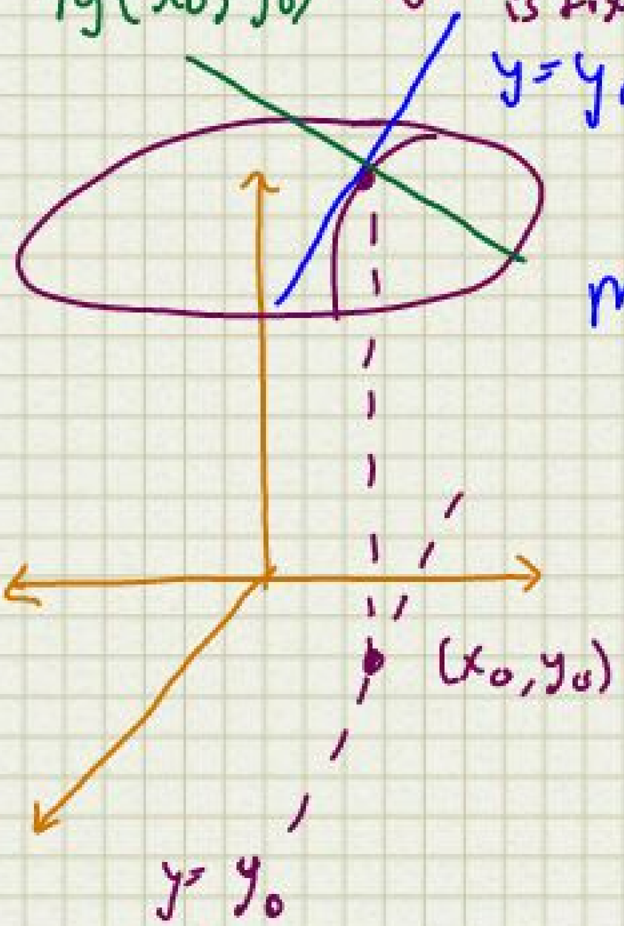
$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

(x direction fixed)

$f_y(x_0, y_0)$ (y direction is fixed)

$y = y_0$



$m_1 = f_x(x_0, y_0)$

$f(x,y) = x^2 + xy + y^3$

$f_x(x,y) = 2x + x + 0$

$f_y(x,y) = 0 + y + 3y^2$

$f_x(1,1) = 2(1) + 1 = 3$

$f(x,y) = e^{\sin(xy^2)}$

$f_x(x,y) = e^{\sin(xy^2)} \cos(xy^2) y^2$

$f_y(x,y) = e^{\sin(xy^2)} \cos(xy^2)\, 2xy$

$f_y(0,\pi) = e^0 \cos(0)\, 0 = 0$

$\boxed{\text{* when deriving for variable treat all other variables as a constant.}}$

$$f(x,y) = \frac{x^2 + y^3}{x+y}$$

$$f_y = \frac{3y^2(x+y) - (1)(x^2+y^3)}{(x+y)^2}$$

(implicit function) Assuming z is Independent

$f(x,y,z)=C \qquad z(x,y)$

z_x

z_y

$x^2+y^2+z^2=4$

$\frac{\partial}{\partial x}\left(x^2+y^2+z^2(x,y)=4\right)$ chain rule

$2x+0+2zz_x=0$

$z_x=\frac{-2x}{2z}=\frac{-x}{z}$

$z=\sqrt{4-x^2-y^2}$

$z_x=\frac{-2x}{2\left(\sqrt{4-x^2-y^2}\right)}=\frac{-x}{\sqrt{4-x^2-y^2}}=\frac{-x}{z}$

$f=(\cos(x))^2$

$f'=2\cos(x)(\cos(x))'$

$=2zz'$

$f=\ln(x^2)$

$f'=\frac{1}{x^2}(x^2)'$

$f=\ln(z)$

$f'=\frac{1}{z}(z)'$

Ex. $\emptyset = xy^2 + xz^3 + \cos(z^2)$

$z_x: \; y^2 + (1)z^3 + x\,3z^2 z_x + (-\sin(z^2))\,2z\,z_x = \emptyset$

$z_x(3xz^2 - 2\sin(z^2)z) = -y^2 - z^3$

$$z_x = \frac{-y^2 - z^3}{3xz^2 - 2\sin(z^2)z}$$

Higher Order Derivatives

$f(x)$

$f'(x)$

$f''(x) = (f'(x))'$

$f(x,y)$

$f_x(x,y)$

$f_y(x,y)$

$f_{xx} = \frac{\partial}{\partial x}(f_x)$

$f_{yy} = \frac{\partial}{\partial y}(f_y)$

$f_{xy} = f_{yx}$

according to theorm

$f_{xy} = \frac{\partial}{\partial y}(f_x)$

$f_{yx} = \frac{\partial}{\partial x}(f_y)$

$f(x,y) = x^3 + x^2y^2 + 3xy + y^4$

$f_{xx} = ?$ $\quad f_x = 3x^2 + 2xy^2 + 3y + \emptyset$ $\quad f_{yy} = ?$ $\quad f_y = \emptyset + 2yx^2 + 3x + 4y^3$

$f_{xy} = f_{yx} = \emptyset + 4xy + 3$

f_{xxx}

f_{xxy}, f_{xyx}, f_{yxx} same f_{xyy}, f_{yxy}, f_{yyx}

$f_{xxx} = 6$

$f_{xxy} = 4y$

$f_{yxx} = 4y$

$$T(x,y,z) = e^{xy^2z}$$

$$T_x = e^{xy^2z}(y^2z) = g_1(x,y,z)$$

$$T_y = e^{xy^2z}(2xyz) =$$

$$T_z = e^{xy^2z}(xy^2)$$

TR = 105 + (TL (12)) %.