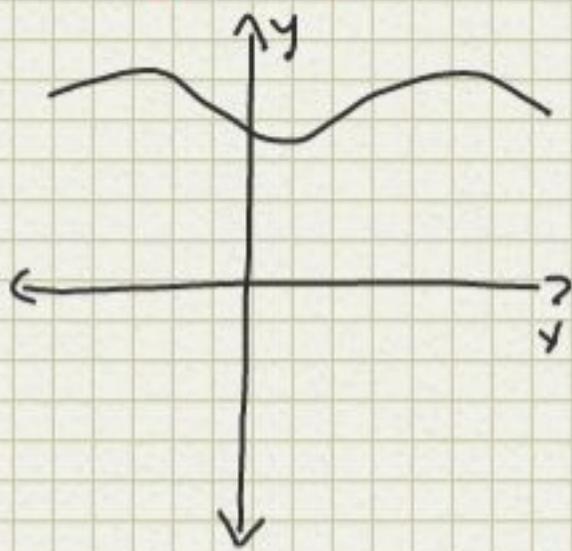


$y(x)$, function in x - y plane



$$k = \frac{|y''(x)|}{(1 + (y'(x))^2)^{3/2}}$$

2/11/2014

$$\vec{T} = \frac{\vec{R}'(t)}{\|\vec{R}'(t)\|} = \vec{g}(t) \quad \vec{T} = \frac{\vec{g}(t)}{g(t)} \quad \vec{T}' =$$

$$R(t) = \langle 4\sin(t), -4\sin(t), 5\cos(t) \rangle$$

$$R'(t) = \langle 4\cos(t), -4\cos(t), -5\sin(t) \rangle$$

$$\|R'(t)\| = \sqrt{16\cos^2(t) + 16\cos^2(t) + 25\sin^2(t)}$$

$$= \sqrt{25\cos^2(t) + 25\sin^2(t) + 7\cos^2(t)}$$

$$= \sqrt{25 + 7\cos^2(t)} \quad \rightarrow 1$$

$$\frac{\|R'(t) \times R''(t)\|}{\|R'(t)\|^3}$$

$$R''(t) = \langle -4\sin(t), 4\sin(t), -5\cos(t) \rangle$$

$$R'(t) \times R''(t) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4\cos(t) & -4\cos(t) & -5\sin(t) \\ -4\sin(t) & 4\sin(t) & -5\cos(t) \end{vmatrix}$$

$$= \hat{i}(20\cos^2(t) + 20\sin^2(t)) - \hat{j}(-20\cos^2(t) - 20\sin^2(t))$$

$$+ \hat{k}(16\sin(t)\cos(t) - 16\sin(t)\cos(t))$$

$$= \langle 20, 20, 0 \rangle$$

$$k = \frac{20\sqrt{2}}{(\sqrt{25 + \cos^2(t)})^3}$$

$$\|R'(t) \times R''(t)\| = 20\sqrt{2}$$

$$y = f(x) \quad \kappa = \frac{|f''(x)|}{(\sqrt{1 + f'(x)^2})^3}$$

$$x = t \quad \vec{R}(t) = \langle t, f(t), 0 \rangle$$

$$y = f(t) \quad \vec{R}'(t) = \langle 1, f'(t), 0 \rangle \quad \|\vec{R}'\| = \sqrt{1 + f'(t)^2}$$

$$z = 0$$

$$\vec{R}''(t) = \langle 0, f''(t), 0 \rangle$$

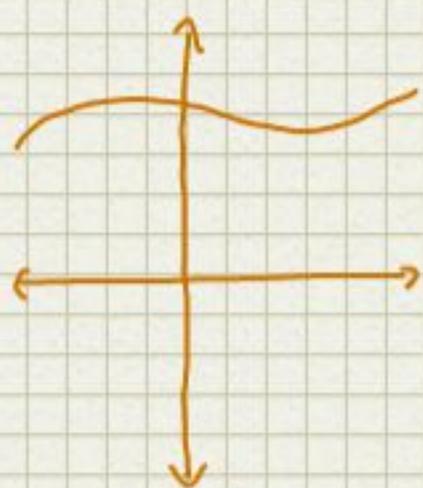
$$\vec{R}'(t) \times \vec{R}''(t) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & f'(t) & 0 \\ 0 & f''(t) & 0 \end{vmatrix} = \langle 0, -0, f''(t) \rangle$$

$$\|\vec{R}' \times \vec{R}''\| = \sqrt{f''(t)^2} = \|f''(t)\|$$

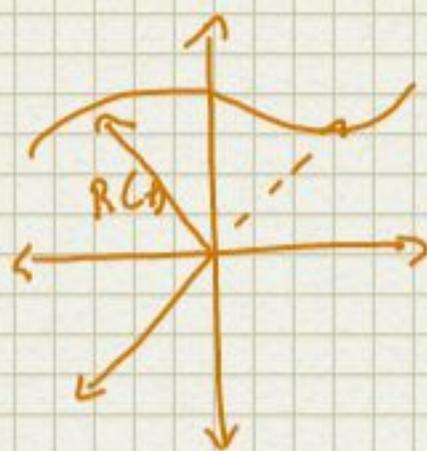
$$\kappa = \frac{f''(t)}{(\sqrt{1 + f'(t)^2})^3} = \frac{f''(x)}{(\sqrt{1 + f'(x)^2})^3}$$

Chapter 11: Function of Several Variables

$$f: \mathbb{R} \rightarrow \mathbb{R}$$



$$f: \mathbb{R} \rightarrow \mathbb{R}^3 \quad \mathbb{R} \rightarrow \mathbb{R}^2$$



Func w/ several variables is a rule that associates a unique value in the range for any (x, y) value in the domain.

$$f: \mathbb{R}^2 \rightarrow \mathbb{R} \quad f = z(x, y)$$

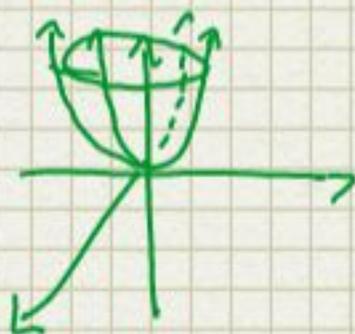
$$3x + 2y + 4z = 1, \quad z = \frac{1}{4} - \frac{3}{4}x - \frac{2}{4}y$$

$$f(z) = \frac{1}{4} - \frac{3}{4}x - \frac{1}{2}y$$

$$f(2, 1) = \frac{1}{4} - \frac{3}{4}(2) - \frac{1}{2}(1) = -7/4$$

$$z = x^2 + y^2 \quad \text{circular paraboloid}$$

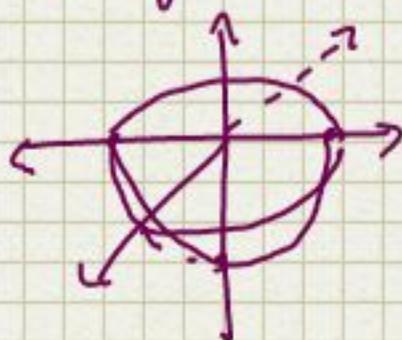
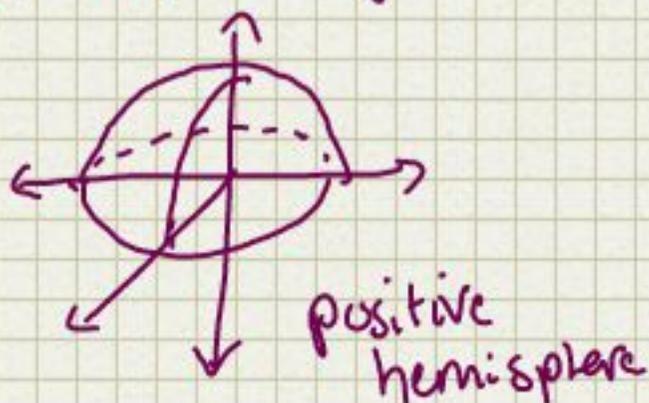
$$f(x, y) = x^2 + y^2$$

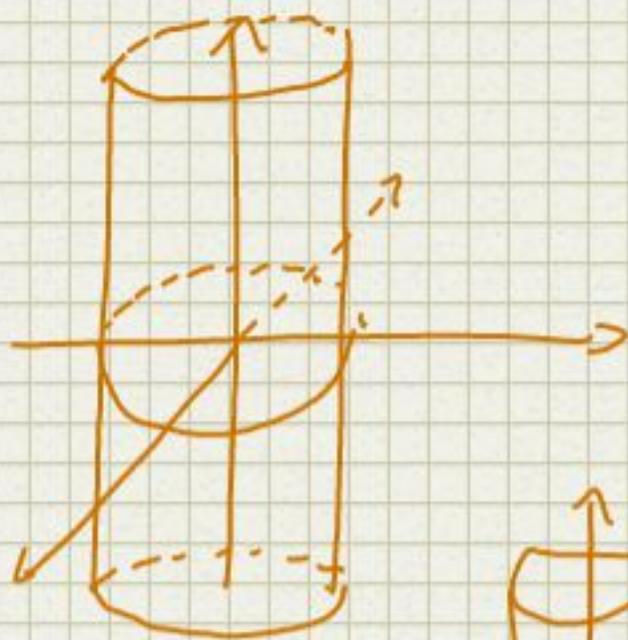


$$x^2 + y^2 + z^2 = 4 \quad \text{implicit function}$$

$$z = \pm \sqrt{4 - x^2 - y^2} \quad (\text{not a function})$$

$$z = \sqrt{4 - x^2 - y^2} \quad z = -\sqrt{4 - x^2 - y^2}$$

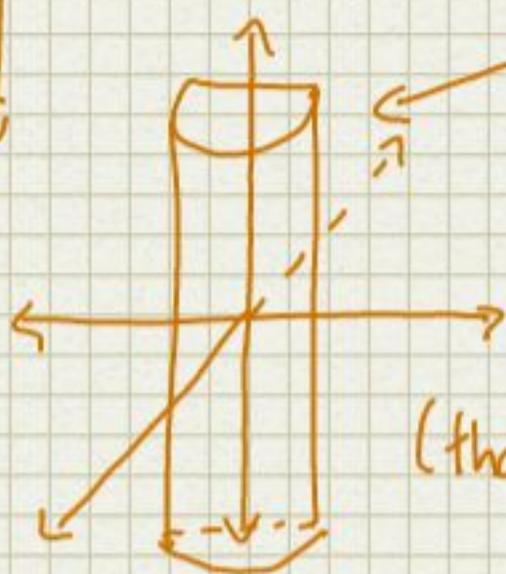




$$x^2 + y^2 = 4$$

cannot solve for z

$$x(y, z) = \pm \sqrt{4 - y^2}$$



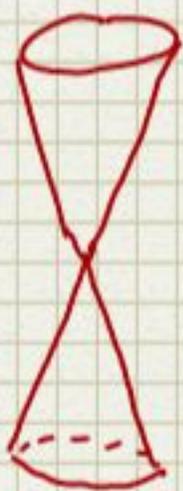
$$x_1(y, z) = \sqrt{4 - y^2}$$

$$x_2(y, z) = -\sqrt{4 - y^2}$$

(the positive half of x -axis)

3D

$$x^2 + y^2 - z^2 = 0$$



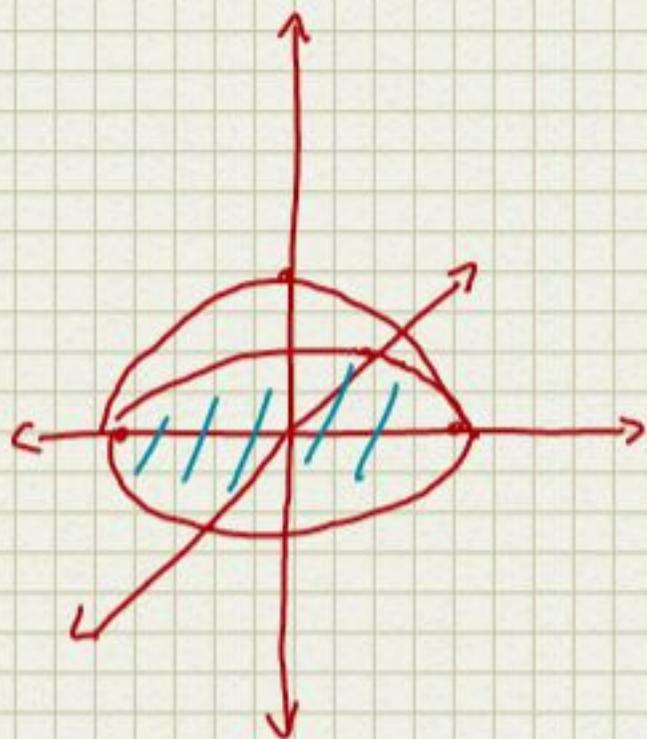
$$z_1 = \sqrt{x^2 + y^2}$$

$$z_2 = -\sqrt{x^2 + y^2}$$

$$f(x,y) = \sqrt{9-x^2-y^2} \quad (\text{positive half of hemisphere w/ radius 3})$$

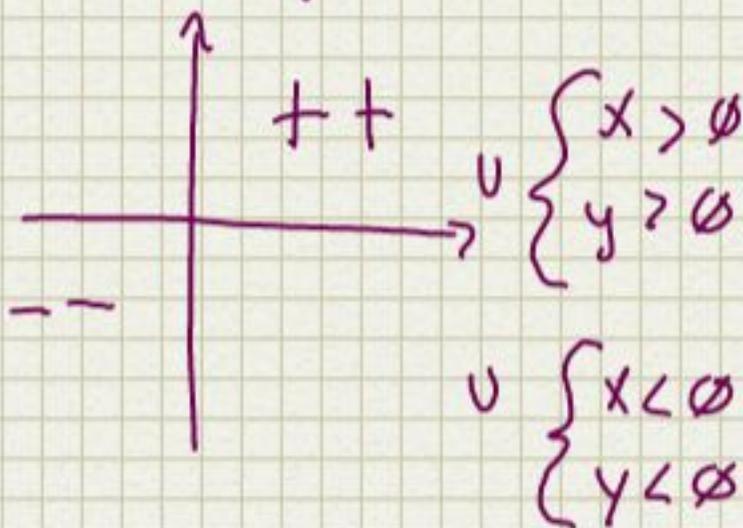
$$\text{Domain: } 9-x^2-y^2 \geq 0 \implies x^2+y^2 \leq 9$$

Circle w/ radius 3



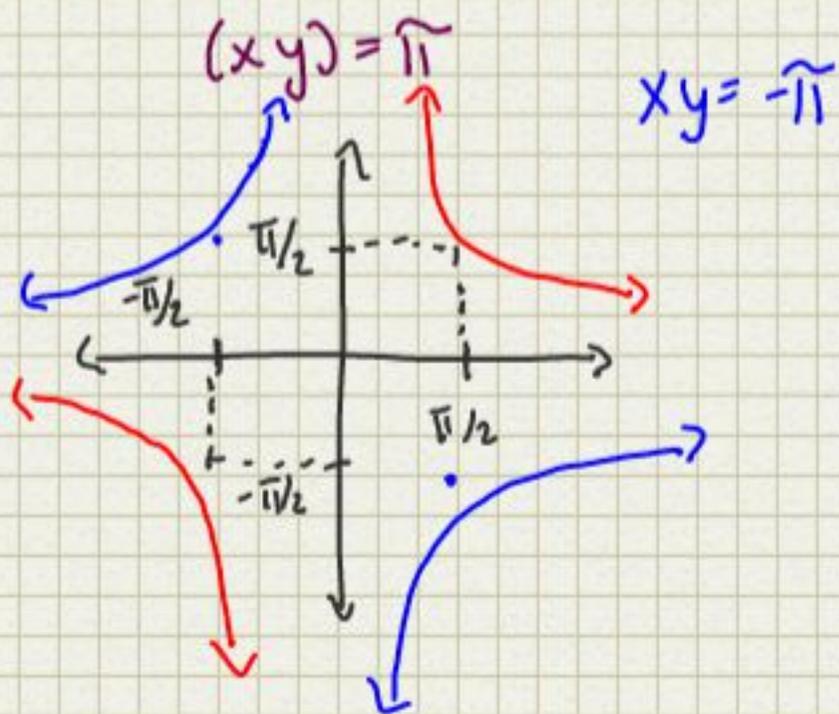
$$f(x,y) = \ln(xy)$$

$$\text{Domain: } xy > 0$$



$$z = \tan(xy)$$

$$xy \neq \pi \pm 2k\pi \quad k=1,2,3,\dots,\infty$$



Function w/ 3 variables

$$T(x, y, z): \mathbb{R}^3 \rightarrow \mathbb{R}$$

$$T(x, y, z, t): \mathbb{R}^4 \rightarrow \mathbb{R}$$

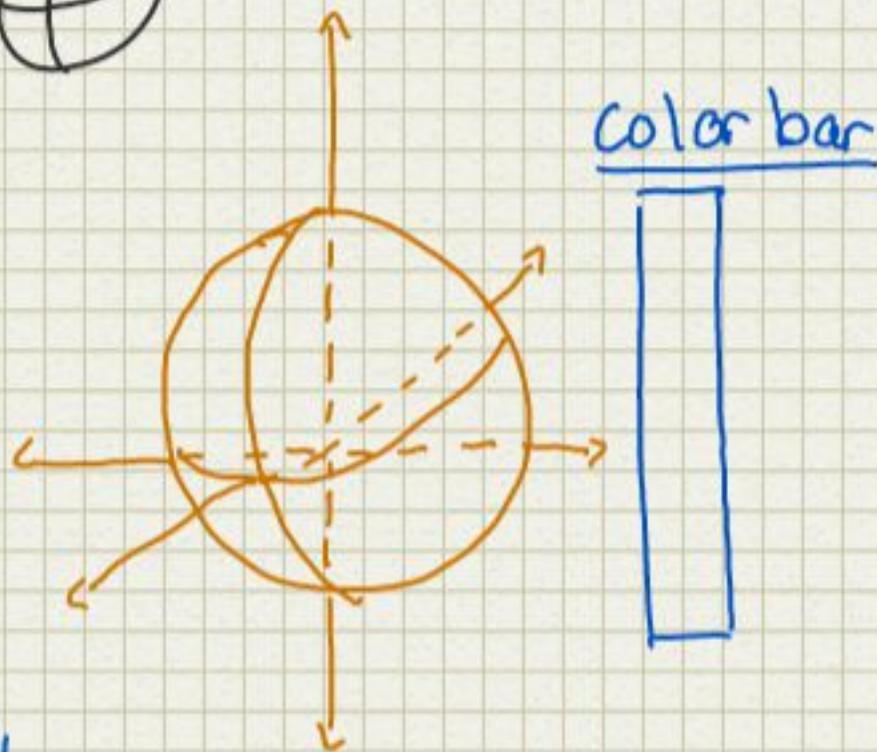
Domain in 3D

Temperature



$$T = (xyz)$$

$$D: x^2 + y^2 + z^2 \leq 9$$



$$x^2 + y^2 + z^2 + T^2 = 4$$

$$T = \pm \sqrt{4 - x^2 - y^2 - z^2} \begin{cases} T_1 = \sqrt{4 - x^2 - y^2 - z^2} \\ T_2 = -\sqrt{4 - x^2 - y^2 - z^2} \end{cases}$$

$$T_1: \begin{aligned} 4 - x^2 - y^2 - z^2 &\geq 0 \\ x^2 + y^2 + z^2 &\leq 4 \end{aligned}$$

$$\underline{f: \mathbb{R}^2 \rightarrow \mathbb{R}}$$

Limit 11.2 $\lim_{(x,y) \rightarrow (x_0,y_0)} f(x,y) = L$

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

$\lim_{x \rightarrow x_0} f(x) = L$ for any $\varepsilon > 0$ there exists a $\delta > 0$ such that for all $x: |x - x_0| \leq \delta$ $x \neq x_0$.

$$|f(x) - L| < \varepsilon$$

Limit in 2D

for any $\varepsilon > 0$ there exists a $\delta > 0$

such that if $(x-x_0)^2 + (y-y_0)^2 \leq \delta^2$ $(x,y) \neq (x_0,y_0)$

then $|f(x,y) - L| < \varepsilon$

