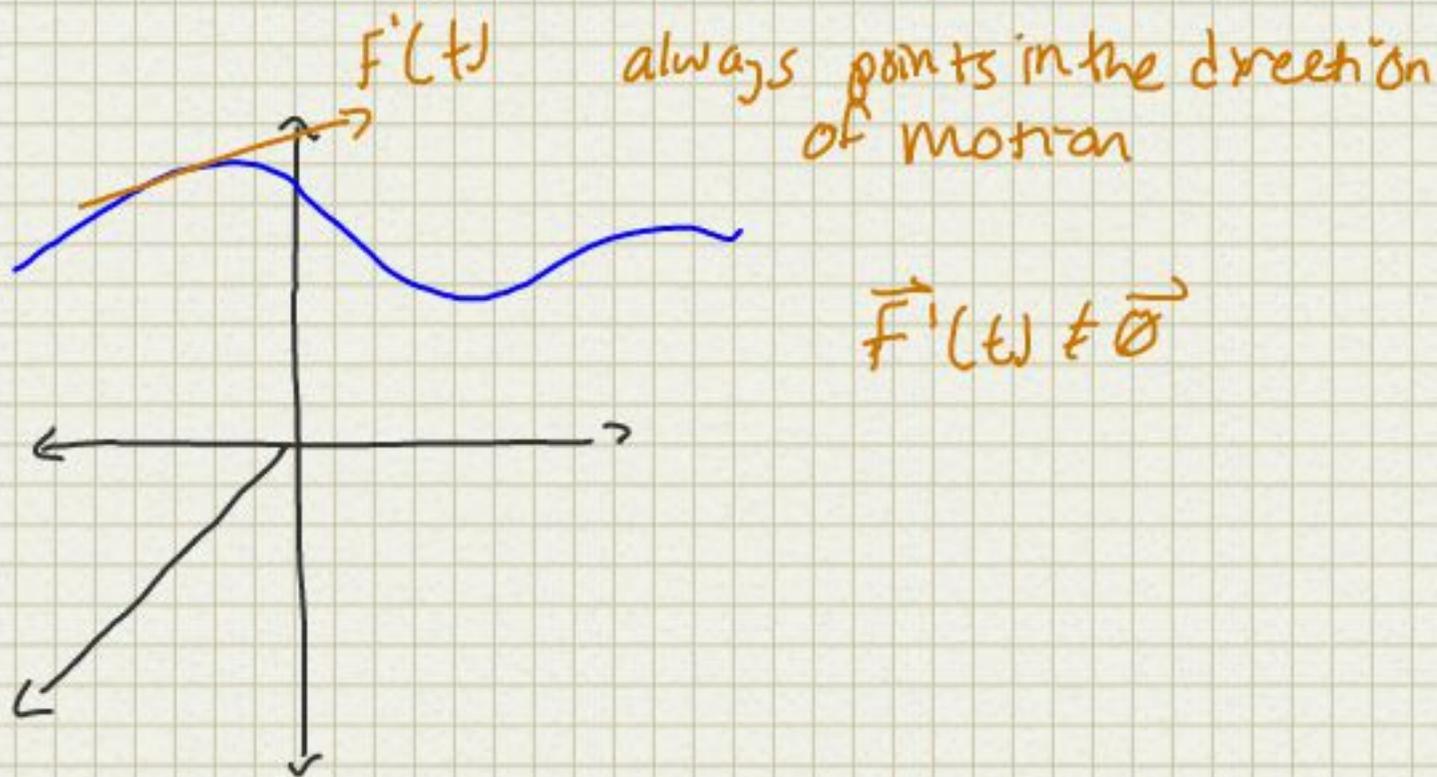


1/30/2014

## Derivatives & Integration of a Vector

$$\vec{F}(t) = \langle x(t), y(t), z(t) \rangle$$

$$\vec{F}'(t) = \lim_{h \rightarrow 0} \frac{F(t+h) - F(t)}{h} = \langle x'(t), y'(t), z'(t) \rangle$$



A v.f.  $\vec{F}(t)$  is smooth at  $t=0$ , so  $\vec{F}'(t)$  exists if  $\neq \vec{0}$

## Second Derivative of Function

$$F''(t) = \frac{d}{dt} (F'(t)) = \frac{d}{dt} \underbrace{\langle x'(t), y'(t), z'(t) \rangle}_{\vec{R}(t)}$$

$$= \vec{R}'(t) = \langle x''(t), y''(t), z''(t) \rangle$$

$$* \frac{d^m \vec{F}(t)}{dt^m} = \langle x^m(t), y^m(t), z^m(t) \rangle$$

$$\vec{F}(t), \vec{G}(t)$$

$$(a\vec{F} \pm b\vec{G})'(t) = a\vec{F}' \pm b\vec{G}'$$

$$(f\vec{F})'(t) = f'\vec{F} + fF'$$

$$(\vec{F} \cdot \vec{G})'(t) = \vec{F}' \cdot \vec{G} + \vec{F} \cdot \vec{G}'$$

$$(\vec{F} \times \vec{G})'(t) = \vec{F}' \times \vec{G} + \vec{F} \times \vec{G}'$$

$$(\vec{F} \cdot \vec{G}) = f_1 g_1 + f_2 g_2 + f_3 g_3$$

$$(\vec{F} \cdot \vec{G})'(t) = (f_1' g_1 + f_1 g_1') + (f_2' g_2 + f_2 g_2') + (f_3' g_3 + f_3 g_3')$$

$$= \langle f_1', f_2', f_3' \rangle \cdot \langle g_1, g_2, g_3 \rangle + \langle f_1, f_2, f_3 \rangle \cdot \langle g_1', g_2', g_3' \rangle$$

$$= \vec{F}' \cdot \vec{G} + \vec{F} \cdot \vec{G}'$$

Ex 1.

$$F(t) = \langle \sin^2(t), e^{3t}, t^2 + 3t \rangle$$

$$F'(t) = \langle 2 \sin(t) \cos(t), 3e^{3t}, 2t + 3 \rangle$$

$$F''(t) = \langle 2 \cos(2t), 9e^{3t}, 2 \rangle$$

$$G(t) = \langle t^2, \ln(t), \tan(t) \rangle \quad G'(t) = \langle 2t, \frac{1}{t}, \sec^2(t) \rangle$$

$$* 2 \sin(t) \cos(t)$$

$$= \sin(2t)$$

$$(\vec{F} \cdot \vec{G})'(t) = \vec{F}' \cdot \vec{G} + \vec{F} \cdot \vec{G}' \quad \begin{array}{l} t > 0 \\ t \neq \frac{\pi}{2} + k\pi \\ t > 0, t \neq \frac{\pi}{2} + k\pi \\ k \text{ positive integer} \end{array}$$

$$= \sin(2t) t^2 + 3e^{3t} \ln(t) + (2t+3) \tan(t) + \sin^2(t) 2t + e^{3t} \cdot \frac{1}{t} + (t^2+3t) \sec^2(t)$$

- If  $\vec{F}(t)$  is smooth and  $\|\vec{F}(t)\| = \text{constant}$  for all  $t$   
then  $\vec{F}(t) \perp \vec{F}'(t)$ .

$$\|\vec{F}(t)\| = \sqrt{x^2(t) + y^2(t) + z^2(t)}$$

$$= \sqrt{\langle x(t), y(t), z(t) \rangle \cdot \langle x(t), y(t), z(t) \rangle}$$

$$= \sqrt{\vec{F}(t) \cdot \vec{F}(t)}$$

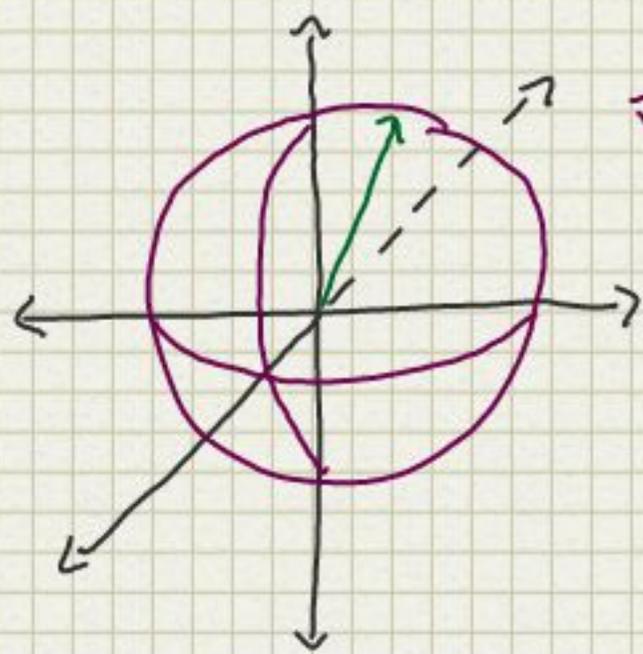
$$\|\vec{F}(t)\|^2 = \vec{F}(t) \cdot \vec{F}(t) = C_1$$

$$(\vec{F}(t) \cdot \vec{F}(t))' = (C_1)'$$

$$F'(t) \cdot F(t) + F(t) \cdot F'(t) = 0$$

$$2 \vec{F} \cdot \vec{F}' = 0$$

$$\|\vec{F}(t)\| = \text{constant}$$



$\vec{v}$  head will always be  
on the surface  
therefore the tangent  
line is perpendicular

$\vec{R}(t)$  description of the object position in the space as a function of parameter Time.

$$R'(t) = \vec{v}(t) : \text{velocity}$$

$$\|R'(t)\| = \|\vec{v}(t)\| : \text{speed}$$

$$R''(t) = \vec{v}'(t) = \vec{A}(t) : \text{Acceleration}$$

Integration

$$\int \vec{F}(dt) = \int \langle x(t), y(t), z(t) \rangle dt$$

$$= \langle \int x(t) dt + \int y(t) dt + \int z(t) dt \rangle + \vec{C}$$

$(C_1, C_2, C_3)$

$$\vec{v}(t) = \int \vec{A}(t) dt + \vec{C}$$

$$\vec{R}(t) = \int \vec{v}(t) dt + \vec{C}_2$$

Ex

$$\vec{A}(t) = \cos(t)\hat{i} - t\sin(t)\hat{k}$$

$$\vec{v}(t) = ? \quad \vec{R}(t) = ?$$

$$\vec{R}(0) = \hat{i} - 2\hat{j} + \hat{k}$$

$$\vec{v}(0) = 2\hat{i} + 3\hat{k}$$

$$\vec{v}(t) = \int \langle \cos(t) + 0 + t\sin(t) \rangle dt + \vec{C}$$

$$\int t\sin(t) = -t\cos(t) + \int \cos(t) dt = -t\cos(t) + \sin(t)$$

$$u = t \quad du = \sin(t) dt$$

$$du = dt \quad v = -\cos(t)$$

$$\vec{v}(t) = \langle \sin(t), 0, -t\cos(t) + \sin(t) \rangle + \langle C_1, C_2, C_3 \rangle$$

$$\vec{v}(0) = \langle 2, 0, 3 \rangle = \langle 0, 0, 0 \rangle + \langle C_1, C_2, C_3 \rangle$$

$$C_1 = 2, \quad C_2 = 0, \quad C_3 = 3$$

$$\vec{v}(t) = \langle \sin(t) + 2, 0, t \cos(t) - \sin(t) + 3 \rangle$$

$$\vec{R}(t) = \int \vec{v}(t) dt + \vec{D} =$$

$$= \int \langle \sin(t) + 2, 0, t \cos(t) - \sin(t) + 3 \rangle dt + \vec{D}$$

$$\int t \cos(t) dt = t \sin(t) - \int \sin(t) dt = t \sin(t) - \cos(t)$$

$$u = t \quad dv = \cos(t) dt$$

$$du = dt \quad v = \sin(t)$$

$$\vec{R}(t) = \langle -\cos(t) + 2t, 0, t \sin(t) + 2 \cos(t) + 3t \rangle + \langle d_1, d_2, d_3 \rangle$$

$$\vec{R}(0) = \langle 1, -2, 1 \rangle = \langle 1, 0, 2 \rangle + \langle d_1, d_2, d_3 \rangle$$

$$1 = 1 + d_1 \quad d_1 = 0$$

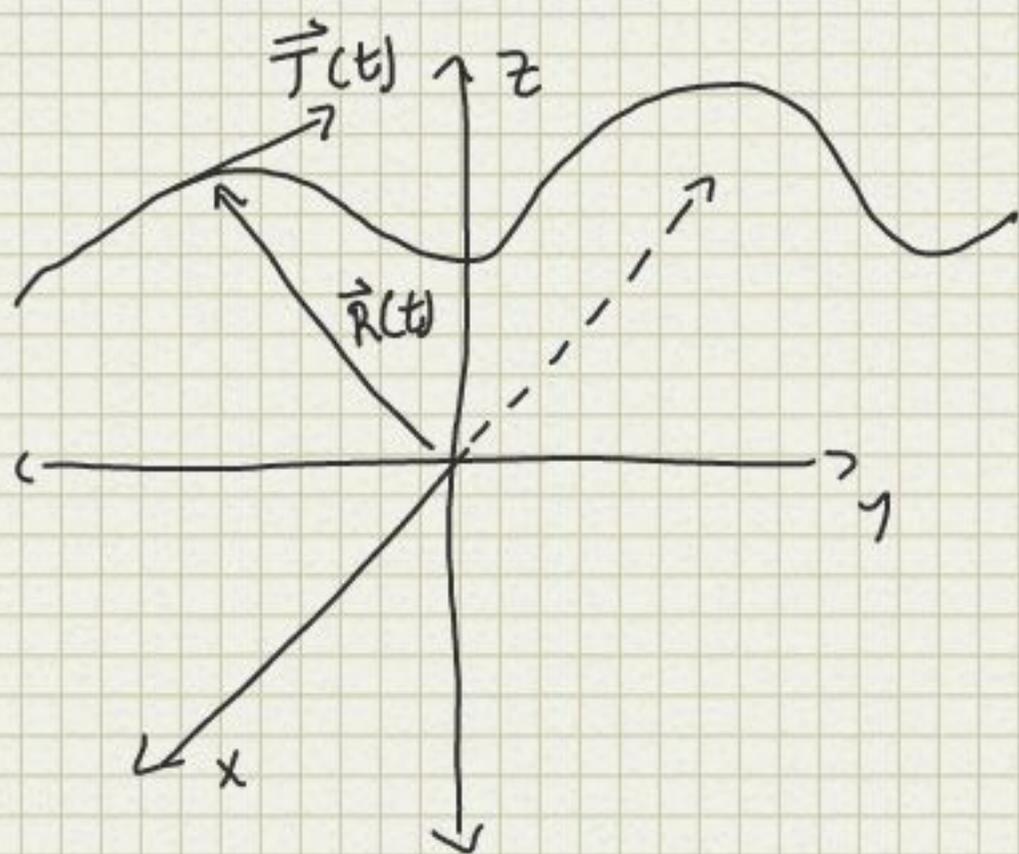
$$-2 = 0 + d_2 \quad d_2 = -2$$

$$1 = 2 + d_3 \quad d_3 = -1$$

$$\vec{R}(t) = \langle -\cos(t) + 2t + 2, -2, t \sin(t) + 2 \cos(t) + 3t - 1 \rangle$$

Assume  $\vec{R}(t)$  is smooth

Find the unit tangent vector  $\vec{T}$  to the curve  $\vec{R}(t)$   
and should point in the direction of motion.



$$\vec{T}(t) = \frac{\vec{R}'(t)}{\|\vec{R}'(t)\|}$$

Test Thursday

Ch 9, 10.1, 10.2