EXAM

Final Exam

Math 3350, Summer 2010

August 6, 2010

- Write all of your answers on separate sheets of paper. You can keep the exam questions when you leave. You may leave when finished.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This exam has 9 problems. There are **420 points total**.

Good luck!

50 pts.

Problem 1. In each part, find the general solution of the differential equation, or solve the given initial value problem. just write down the answer.

A.

$$\frac{dy}{dx} = \frac{4y}{1+2x}$$

B.

$$\frac{dy}{dx} + 3y = x^4 e^{-3x}$$

C.

$$\frac{dy}{dx} + 2y = e^{2x} y^{-1/2}$$

D.

$$\frac{dy}{dx} = (2x + y + 1)^2 - 2$$

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60 pts.

Problem 2. Newton's law of cooling says that the time rate of change dT/dt of the temperature T of a body is proportional to the difference between T and the temperature M of the surrounding medium (the temperature of the surrounding medium is assumed to stay constant).

A soda can at a temperature of 35° is placed in a room that is at 75°. After 10 minutes the can has warmed to a temperature of 55°.

A. Find the differential equation for the temperature T of the can and solve it to find T as a function of time.

B. What is the temperature of the can after 15 minutes? Give a numerical answer accurate to two decimal places.

C. At what time will the temperature of the coffee be 74°? Give an numerical answer that is accurate to two decimal places.

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50 pts.

Problem 3. In each part, find the general solution, or solve the initial value problem.

A.

$$y'' - 3y' + 2y = 0$$

B.

$$y'' + 2y' + y = 0, \qquad y(0) = 1, \quad y'(0) = -3$$

C.

$$y'' + 4y' + 13y = 0$$

D.

$$x^2y'' - 3xy' + 4y = 0$$

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40 pts.

Problem 4. **Use the method of Undetermined Coefficients** to find the general solution. No credit for doing it with a different method.

A.

$$y'' - 3y' + 2y = 2x^2 + 2x$$

B.

$$y'' + y' - 6y = e^{2x}$$

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40 pts.

Problem 5. Find the general solution **by the method of variation of parameters**. No credit for doing it by a different method.

$$y'' - 2y' + y = xe^x$$

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60 pts.

Problem 6. In each part, find the inverse Laplace Transform.

A. In this part, do the partial fractions decomposition by hand.

$$\frac{4+s-2s^2}{s(s-1)(s-2)}$$

B. In this part, you can use a calculator to do the partial fractions

$$\frac{3s^2+15s+16}{(s+1)(s+2)^2}$$

C.

$$\frac{2s-1}{s^2-2s+5}$$

(Hint: complete the square in the denominator.)

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40 pts.

Problem 7. Find the Laplace Transform of the function

$$f(t)=\begin{cases}0, & 0<t<1\\ t^2, & 1<t<2\\ 0, & 2<t<\infty\end{cases}$$

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40 pts.

Problem 8. Find the Inverse Laplace Transform of the following function:

$$F(s)=\frac{1}{s+2}+e^{-2s}\frac{s+1}{s^2+4}$$

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40 pts.

Problem 9. Solve the following initial value problems **by the method of Laplace Transforms**.

A.

$$y''-y=e^t, \qquad y(0)=1, \quad y'(0)=0.$$

B.

$$y''+y=\mathscr{U}(t-1), \qquad y(0)=1, \quad y'(0)=0$$
