
PROBLEM SET

Assignment 1

Math 3310-101, Spring 2015

June 5, 2015

- Write all of your answers on separate sheets of paper. You can keep the question sheet.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This problem set has 5 problems. There are **500 points total**.

Good luck!

100 pts. **Problem 1.** Show by induction that

$$1^2 + 2^2 + \cdots + n^2 = \sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6},$$

for all natural numbers n .

100 pts. **Problem 2.** Show by induction that

$$2^n < n!$$

for all natural numbers $n \geq 4$.

100 pts. **Problem 3.** Find the largest natural number m so that $n^3 - n$ is divisible by m for all $n \in \mathbb{N}$. Prove that your answer is correct.

100 pts. **Problem 4.** Let $a = 2079$ and $b = 5733$. Use the Euclidean Algorithm to find (a, b) . Find integers s and t so that

$$(a, b) = sa + tb.$$

100 pts. **Problem 5.** Do Exercise 10 in in 3.10, with the following clarifications.
Since you can't see the pennies, you don't know which are showing heads and which are showing tails. You can ask the villain to flip some pennies. It would be cheating to make the flip depend on which face a penny is showing, but you can tell him to flip all of the coins in a particular group. (Ask me if further clarification is needed.)
