Math 3350-202
Summer II, 2012 Exam 3

Instructions.

This is a take-home exam.

You can use the book, your notes and the handouts on my website

You can discuss the problems with your classmates, but write up your own solutions.

Write your answers on seperate paper, not on the questions handout.

You must show enough work to justify your answers. Give exact answers, not approximations (e.g. $\sqrt{2}$, not 1.414). You may use a TI-89 Calulator to do integrals and partial fractions. Indicate where you used the calculator.

There are 7 problems and 300 points total.

① Use the method of undetermined coefficients to solve the differential equation

40pts $$y'' - 3y' + 2y = xe^{2x}$$

Give the general solution.

② Use the method of variation of parameters to solve the inhomogeneous Euler-Cauchy equation

40pts $$x^2y'' - 2y = x^2.$$

Give the general solution, (Hint: you have to put the equation is standard form before applying variation of parameters!)

③ In each part, find the inverse Laplace transforms.

60pts (A) In this part, find the partial fractions decomposition by hand!

$$F(s) = \frac{2s^3 - 4s^2 - 3s + 2}{(s-1)^2 s^2}$$

(3B) In this part, use a calculator to find the partial fractions decomposition.

$$F(s) = \frac{3s^4 + 3s^3 + 2s^2 - 7s + 5}{(s+1)^3 (s^2+4)}$$

(C)

$$F(s) = \frac{s+5}{s^2 - 4s + 13}$$

Hint: Complete the square in the denominator.

(4) Solve the following Initial value problems by the method of Laplace Transforms. 40pts

(A) $y'' + 4y = \sin(2t)$, $y(0) = 1$, $y'(0) = 0$

(B) $y'' + 2y' + y = t^2 e^{-t}$, $y(0) = 1$, $y'(0) = -2$

(5) Find the Laplace Transform of the function $f(t)$ defined by

40pts $$f(t) = \begin{cases} t, & 0 < t < 1 \\ t^2, & 1 < t < 2 \\ 1, & 2 < t < \infty \end{cases}$$

(6) Find the Inverse Laplace Transform of $F(s)$:

40pts $$F(s) = e^{-2s}\frac{s}{s^2+4} + e^{-3s}\frac{1}{(s-2)^2}$$

(7) Solve the initial value problem

40pts $$y'' + 4y = \mathcal{U}(t-2)$$
$$y(0) = 1$$
$$y'(0) = 2$$