PROBLEM SET

Problems on Span, Independence, and Matrix Spaces

Math 3351, Fall 2010

Sept. 27, 2010

- Write all of your answers on separate sheets of paper. You can keep the question sheet.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This problem set has 8 problems.

Good luck!

Problem 1. Consider the matrix

$$A = \begin{bmatrix} -15 & -80 & -145 & 22 & -181 \\ -89 & -19 & 51 & 50 & 132 \\ 66 & -62 & -190 & 78 & -96 \\ 77 & 81 & 85 & -8 & 150 \end{bmatrix}.$$

A Find a basis of the nullspace of A.

B Find a basis of the rowspace of A.

C Find a basis of the column space of A. Write the other columns of A as linear combinations of these basis vectors.

D What is the rank of A?

Problem 2. Consider the matrix

$$A = \begin{bmatrix} 8 & 9 & 19 & 4 & 22 & -25 \\ 7 & 1 & -4 & 6 & 3 & 10 \\ 5 & 5 & 10 & 2 & 13 & -12 \\ 2 & 7 & 19 & 4 & 12 & -33 \\ 4 & 4 & 8 & 6 & 6 & -14 \\ 4 & 1 & -1 & 6 & 0 & 1 \end{bmatrix}$$

A Find a basis of the nullspace of A.

B Find a basis of the rowspace of A.

C Find a basis of the columnspace of A. Write the other columns of A as linear combinations of these basis vectors.

D What is the rank of A?

---

r

Problem 3. Consider the vectors

$$v_1 = \begin{bmatrix} 5 \\ 6 \\ 2 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 7 \\ 8 \\ 1 \\ 4 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 11 \\ 12 \\ -1 \\ 12 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 12 \\ 14 \\ 3 \\ 4 \end{bmatrix}, \quad v_5 = \begin{bmatrix} 3 \\ 3 \\ 8 \\ 1 \end{bmatrix}.$$

Let $S = \text{span}(v_1, v_2, v_3, v_4, v_5)$, which is a subspace of $\mathbb{R}^4$.

A. Find a basis of S. What is the dimension of S?

B. Express the vectors in the list $v_1, \ldots, v_5$ that are not part of the basis as linear combinations of the basis vectors.

C. Consider the vectors

$$w_1 = \begin{bmatrix} 7 \\ 9 \\ 0 \\ 3 \end{bmatrix}, \quad w_2 = \begin{bmatrix} 12 \\ 13 \\ 27 \\ -1 \end{bmatrix}.$$

Determine if these vectors are in S. If the vector is in S, express it as a linear combination of the basis vectors found above.

Problem 4. Consider the vectors

$$v_1 = \begin{bmatrix} 5 \\ 6 \\ 2 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 7 \\ 8 \\ 1 \\ 4 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 11 \\ 12 \\ -1 \\ 12 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 12 \\ 14 \\ 3 \\ 4 \end{bmatrix}, \quad v_5 = \begin{bmatrix} 3 \\ 3 \\ 8 \\ 1 \end{bmatrix}.$$

Let $S = \text{span}(v_1, v_2, v_3, v_4, v_5)$, which is a subspace of $\mathbb{R}^5$.

A. Find a basis of S. What is the dimension of S?

B. Express the vectors in the list $v_1, \ldots, v_5$ that are not part of the basis as linear combinations of the basis vectors.

C. Consider the vectors

$$w_1 = \begin{bmatrix} 7 \\ 9 \\ 0 \\ 3 \end{bmatrix}, \quad w_2 = \begin{bmatrix} 12 \\ 13 \\ 27 \\ -1 \end{bmatrix}$$

Determine if these vectors are in S. If the vector is in S, express it as a linear combination of the basis vectors found above.

Problem 5. Determine if the following vectors are independent or dependent. If they are independent, complete the list to a basis of $\mathbb{R}^5$ by adding on a standard basis vector. If the vectors are dependent, find a linear relation between them.

$$v_1 = \begin{bmatrix} 13 \\ -10 \\ 24 \\ -9 \\ -11 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 4 \\ -1 \\ 1 \\ -2 \\ 0 \end{bmatrix}, \quad v_3 = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 2 \\ 0 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 1 \\ -2 \\ 8 \\ 0 \\ -4 \end{bmatrix}$$

Problem 6. Determine if the following vectors are independent or dependent. If they are independent, complete the list to a basis of $\mathbb{R}^5$ by adding on a standard basis vector. If the vectors are dependent, find a linear relation between them.

$$v_1 = \begin{bmatrix} -38 \\ 91 \\ -1 \\ 63 \\ -23 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -63 \\ -26 \\ 30 \\ 10 \\ 22 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 12 \\ 45 \\ -14 \\ 60 \\ -35 \end{bmatrix}, \quad v_4 = \begin{bmatrix} -89 \\ -170 \\ 64 \\ 24 \\ 42 \end{bmatrix}$$

Problem 7. Consider the vectors

$$v_1 = \begin{bmatrix} 4 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 4 \\ 5 \\ 3 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 4 \\ -1 \\ -3 \\ 1 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 28 \\ 26 \\ 12 \\ 7 \end{bmatrix},$$

$$v_5 = \begin{bmatrix} 4 \\ 5 \\ 1 \\ 3 \end{bmatrix}, \quad v_6 = \begin{bmatrix} 0 \\ 6 \\ 0 \\ 6 \end{bmatrix}$$

Do these vectors span $\mathbb{R}^4$? If so, select a basis of $\mathbb{R}^4$ out of this list of vectors.

Problem 8. Consider the vectors

$$v_1 = \begin{bmatrix} 3 \\ 6 \\ 5 \\ 8 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 9 \\ 4 \\ 2 \\ 4 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 3 \\ -8 \\ -8 \\ -12 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 3 \\ 0 \\ 7 \\ 2 \end{bmatrix},$$

$$v_5 = \begin{bmatrix} 9 \\ 10 \\ 0 \\ 10 \end{bmatrix}, \quad v_6 = \begin{bmatrix} 3 \\ 4 \\ 8 \\ 0 \end{bmatrix}, \quad v_7 = \begin{bmatrix} 42 \\ 22 \\ 47 \\ 8 \end{bmatrix}$$

Do these vectors span $\mathbb{R}^4$? If so, select a basis of $\mathbb{R}^4$ out of this list of vectors.
