EXAM

Exam 3, Take-home Exam

Math 3350, Spring 2009

April 20, 2009

- This is a Take-home exam, due on Thursday, April 30.
- Write all of your answers on separate sheets of paper. You can keep the exam questions. You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- Unless otherwise instructed, you can use a calculator to do integrals and partial fractions decompositions. State clearly what you are going to put into the calculator and what you got out.
- You can use the textbook and your notes. You can discuss the problems with other people, but write up your own answers, don't just copy from someone else.
- This exam has 8 problems. There are **340 points total**.

Good luck!

40 pts.

Problem 1. **Use the Shifting Rule version of the method of undetermined coefficients** to find the general solution. **No credit** for using another method (including the book's version of undetermined coefficients). If you're wondering what the method is, or need help with it, see `http://www.math.ttu.edu/~drager/Expos/shiftrule.pdf`.

A.

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 6y = (t^2 - t)e^t.$$

B.

$$\frac{d^2y}{dt^2} + y = t\sin(t).$$

---

60 pts.

Problem 2. In each part, find the inverse Laplace Transform.

A. In this part, do the partial fractions decomposition by hand.

$$\frac{2s^2 - 7s + 3}{s^2(s-1)^2}.$$

B. In this part, you can use a calculator to do the partial fractions decomposition

$$\frac{3\,s^5 - s^3 - s^4 + 12\,s^2 - 20\,s + 12}{s^2\,(s-1)^2\,(s^2+4)}.$$

C.

$$\frac{s}{s^2 - 2s + 5}.$$

(Hint: complete the square in the denominator.)

---

40 pts.

Problem 3. Solve the following initial value problems **by the method of Laplace Transforms**.

A.

$$y'' + y = e^{2t}, \qquad y(0) = 1, \quad y'(0) = 1.$$

B.

$$y'' - 4y' + 4y = e^{2t}, \qquad y(0) = 1, \quad y'(0) = -1$$

---

40 pts.

Problem 4. Find the Laplace Transform of the function

$$f(t) = \begin{cases} t, & 0 < t < 2 \\ 0, & 2 < t < 3 \\ t^2, & 3 < t < \infty. \end{cases}$$

---

40 pts.

Problem 5. Find the Inverse Laplace Transform of the following function:

$$F(s) = \frac{1}{s} + e^{-s}\frac{1}{(s-1)(s-2)} + e^{-5s}\frac{1}{s^2+4}.$$

---

40 pts.

Problem 6. **Use Laplace Transforms** to solve the following initial value problem.

$$y'' + 4y = u(t-1)t, \qquad y(0) = 1, \quad y'(0) = 0.$$

---

40 pts.

Problem 7. Find the following convolutions directly from the definition. Compute the integrals by hand.

A. $e^{2t} * e^{3t}$.

B. $t * t^2$.

---

40 pts.

Problem 8. **Use Laplace Transforms** to find the convolution

$$e^{2t} * \cos(t).$$

You can use a calculator to find the partial fractions decomposition.
