
EXAM

Exam 3
Take Home Exam

Math 3350, Summer 2008

August 1, 2008

- This is a Takehome Exam. You may discuss the problems with others, but write up your own solutions.
- If not otherwise instructed, you can use a calculator to do the integrals, but state exactly what you used the calculator to compute.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This exam has 8 problems. There are **380 points total**.

Good luck!

40 pts.

Problem 1. Find the Laplace Transform of the given function.

A.

$$f(t) = t^4 - 2t^3 + t^2 - 3t + 2$$

B.

$$f(t) = 5e^{-t} + 2e^{3t} + t^2 e^{5t}$$

C.

$$f(t) = 3e^{-2t} \sin(5t) + 4e^{-2t} \cos(5t)$$

D.

$$f(t) = te^{-3t} \cos(5t)$$

40 pts.

Problem 2. In each part, give the **form** of the partial fractions decomposition, with undetermined coefficients. **Do not find the coefficients.** No computation is required.

A.

$$\frac{s^2}{(s-1)(s-2)(s+2)}$$

B.

$$\frac{s^4 + 1}{s^2(s-1)^3(s-3)}$$

C.

$$\frac{s^2 + s + 1}{(s-1)(s^2 + 1)}$$

D.

$$\frac{s^9 + 5}{s^2(s^2 + 1)^3}$$

80 pts.

Problem 3. In each part, find the inverse Laplace Transform. (Unless otherwise stated, you can use a calculator to find the partial fractions decomposition.)

A. In this part, find the partial fractions decomposition by hand.

$$F(s) = \frac{5s^2 - 3s + 1}{s^2(s-1)}$$

B.

$$F(s) = \frac{5s^4 - 15s^3 + 15s^2 - 7s + 1}{(s-1)^3 s^3}$$

C.

$$F(s) = \frac{5s - 7}{s^2 - 6s + 13}$$

D.

$$F(s) = \frac{s^2}{(s^2 + 1)^2}$$

40 pts.

Problem 4. Solve the following initial value problems by the method of Laplace Transforms.

A.

$$y'' - 3y' + 2y = t^2 + e^t, \quad y(0) = 1, \quad y'(0) = 1$$

B.

$$y'' + 4y = \sin(2t), \quad y(0) = 1, \quad y'(0) = 0$$

40 pts.

Problem 5. Find the Laplace Transform of the function

$$f(t) = \begin{cases} t, & 0 \leq t < 1, \\ t^2, & 1 \leq t < 2 \\ 1, & 2 < t < \infty \end{cases}$$

40 pts.

Problem 6. Let $f(t)$ be defined by

$$f(t) = \begin{cases} 0, & 0 < t < 1 \\ t, & 1 < t < 2 \\ 0, & 2 < t < \infty. \end{cases}$$

Use **Laplace Transforms** to solve the initial value problem

$$y'' + y = f(t), \quad y(0) = 0, \quad y'(0) = 1.$$

60 pts.

Problem 7. Find the following convolutions directly from the definition. You can use a calculator for the integrals.

A. $t^2 * t^3$

B. $e^{2t} * e^{-t}$

C. $\sin(t) * \sin(t)$.

40 pts.

Problem 8. In each part, use **Laplace Transforms** to find the convolution.

A. $e^t * \cos(2t)$

B. $t^2 * \sin(3t)$
