EXAM

Exam 1
Revised for Posting

Math 3350, Summer 2008

July 18, 2008

- Write all of your answers on separate sheets of paper. You can keep the exam questions when you leave. You may leave when finished.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This exam has 5 problems. There are **270 points total**.

Good luck!

140 pts.

Problem 1. In each part, find the general solution of the differential equation, or solve the given initial value problem. You must show the steps in solving the equation by one of the methods given in class, you can't just write down the answer.

A.

$$\frac{dy}{dx} = 4x^3y^4$$

B.

$$\frac{dy}{dx} = \frac{3y}{x+1}, \qquad y(0) = 1$$

C.

$$\frac{dy}{dx} + 3y = x^3e^{-3x}$$

D.

$$\frac{dy}{dx} + \frac{2x}{1+x^2}y = x, \qquad y(0) = 5$$

E.

$$\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$$

F.

$$\frac{dy}{dx} + 2y = e^{5x}y^{3/2}$$

G.

$$\frac{dy}{dx} = (2x + y)^2 - 2$$

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0 pts.

Problem 2. **This problem omitted.**

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40 pts.

Problem 3. Find an integrating factor that depends on only one of the variables and use it to solve the equation

$$(y + 1)\,dx + (2x + xy)\,dy = 0.$$

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40 pts.

Problem 4. A tank contains 100 gallons of water. Two gallons of brine per minute flow into the tank, each gallon of brine containing 1/2 pound of salt. Two gallons of water flow out of the tank per minute. Assume that the tank is kept well stirred.

Find a differential equation for the number of pounds of salt in the tank (call it y, say).

Assuming the tank intially contains 10 pounds of salt, solve this differential equation.

At what time will there be 23 lbs of salt in the tank? Give a numerical answer accurate to two decimal places.

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50 pts.

Problem 5. Newton's law of cooling says that the time rate of change dT/dt of the temperature T of a body is proportional to the difference between T and the temperature M of the surrounding medium (the temperature of the surrounding medium is assumed to stay constant).

A cup of coffee at a temperature of 210° is placed in a room that is at 70°. After 10 minutes the coffee has cooled to a temperature of 150°.

A. Find the differential equation for the temperature T of the coffee and solve it to find T as a function of time.

B. What is the temperature of the coffee after 15 minutes? Give a numerical answer accurate to two decimal places.

C. At what time will the temperature of the coffee be 72°? Give an numerical answer that is accurate to two decimal places.
