
EXAM

Exam 3
Take Home Exam

Math 2350-02, Summer II 2008

August 1, 2008

- This is a Takehome Exam. You may discuss the problems with others, but write up your own solutions.
- If not otherwise instructed, you can use a calculator to do the integrals, but state exactly what you used the calculator to compute.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This exam has 8 problems. There are **360 points total**.

Good luck!

40 pts. **Problem 1.** Find the critical points of the function

$$f(x, y) = x^2 + y^3 + 6xy + 24y.$$

Classify each of the critical points as a relative maximum, relative minimum, or a saddle point.

40 pts. **Problem 2. Use Lagrange Multipliers** to find the max and min of the function $f(x, y) = xy + z$ on the sphere $x^2 + y^2 + z^2 = 3$.

60 pts. **Problem 3.** Let D be the region in the xy -plane bounded by the curve $y = 1 - x^2$ and the line $x + y = 1$.

A. Find an iterated integral for computing

$$\iint_D y \, dA$$

where the order of integration is $dy \, dx$ (i.e., integrate first with respect to y and then with respect to x). Don't evaluate the integral yet.

B. Find an iterated integral for computing

$$\iint_D y \, dA$$

where the order of integration is $dx \, dy$ (i.e., integrate first with respect to x and then with respect to y). Don't evaluate the integral yet.

C. Evaluate one of the integrals above by hand computation.

40 pts. **Problem 4.** Consider the iterated integral

$$\int_0^2 \int_0^{x^2} f(x, y) \, dy \, dx.$$

Sketch the region of integration and find an equivalent integral with the order of integration reversed.

40 pts. **Problem 5.** Let D be the region in the first quadrant bounded by the coordinate axes and the circle $x^2 + y^2 = a^2$. Find the centroid of D , and the moment of inertia of D for rotation about the y -axis.

60 pts.

Problem 6. Let D be the solid in the first octant bounded by the coordinate planes and the plane $2x + 2y + z = 2$.

- A. Find an iterated integral for calculating the volume of D where the first integration is with respect to z . Don't evaluate the integral yet.
 - B. Find an iterated integral for calculating the volume of D where the first integration is with respect to y . Don't evaluate the integral yet.
 - C. Evaluate one of the integrals above by hand computation.
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40 pts.

Problem 7. Let D be the solid bounded by the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 2$. Use **cylindrical coordinates** to evaluate

$$\iiint_D x^2 z \, dV.$$

40 pts.

Problem 8. Let D be the solid bounded by the sphere $x^2 + y^2 + z^2 = a^2$. Use **spherical coordinates** to evaluate the integral

$$\iiint_D z^2 \, dV.$$
