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## PROBLEM SET

Practice Lagrange Multiplier Problems

Math 2350, Spring 2008

March 14, 2008

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- These are practice problems (don't turn them in) with (partial) answers provided.

Good luck!

**Problem 1.** Find the maximum and minimum of the function  $f(x, y) = xy^2$  on the circle  $x^2 + y^2 = 1$ . Since the circle is a closed bounded curve, the maximum and minimum exist.

Answer: the critical points are

$$(x, y) = (\pm 1, 0) \quad (x, y) = (\pm\sqrt{3}/3, \sqrt{6}/3) \\ (x, y) = (\pm\sqrt{3}/3, -\sqrt{6}/3)$$

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**Problem 2.**

Find the max and min of the function  $f(x, y, z) = xy + yz$  on the sphere  $x^2 + y^2 + z^2 = 1$ . Since the sphere is a closed bounded surface, the max and min exist.

Answer: Critical points  $(x, y, z)$ ,

$$\begin{aligned} & ( (1/2)\sqrt{2}, 0, -1/2\sqrt{2} ) \\ & ( -(1/2)\sqrt{2}, 0, 1/2\sqrt{2} ) \\ & ( 1/2, (1/2)\sqrt{2}, 1/2 ) \\ & ( 1/2, -(1/2)\sqrt{2}, 1/2 ) \\ & ( -1/2, -(1/2)\sqrt{2}, -1/2 ) \\ & ( -1/2, (1/2)\sqrt{2}, -1/2 ) \end{aligned}$$

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**Problem 3.**

A box is to be constructed with a volume of 1372 cubic inches. The box has 4 sides and a bottom, but no top. What are the dimensions of the cheapest box?

Answer: Height 7, other dimensions 14.

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**Problem 4.**

Find the max and min of the function  $f(x, y) = x^2 - x/2 + y^2 - y$  on the disc enclosed by the unit circle  $x^2 + y^2 = 1$ .

Answers: There is one interior critical point at  $(1/4, 1/2)$ , which is the minimum. Using Lagrange multipliers, there are two critical points on the boundary

circle, namely  $(\sqrt{5}/5, 2\sqrt{5})$  and  $(\sqrt{5}/5, 2\sqrt{5})$ . The second of these is the global maximum.

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**Problem 5.**

Let  $C$  be the curve of intersection of the cylinder  $x^2 + y^2 = 1$  and the plane  $x + y + z = 1$ . Find the highest and lowest points on the curve, i.e. maximize and minimize the function  $f(x, y, z) = z$  over  $C$ .

Answer:  $(\sqrt{2}/2, \sqrt{2}/2, \sqrt{2})$  and  $(-\sqrt{2}/2, -\sqrt{2}/2, -\sqrt{2})$

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