

---

# EXAM

Exam 2  
Takehome Exam

Math 2350-02&04, Spring 2008

April 23, 2008  
Corrected Version April 28, 2008

---

- This is a Takehome Exam. You may discuss the problems with others, but write up your own solutions.
- If not otherwise instructed, you can use a calculator to do the integrals, but state exactly what you used the calculator to compute.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g.,  $\sqrt{2}$ , not 1.414).
- This exam has 8 problems. There are **340 points total**.

Good luck!

40 pts. **Problem 1.** Let  $R$  be the region in the upper half-plane bounded by the circles  $x^2 + y^2 = 1$  and  $x^2 + y^2 = 9$ . Find  $\bar{y}$ , the  $y$ -coordinate of the centroid of  $R$ .

---

60 pts. **Problem 2.** Let  $R$  be the region in three dimensional which lies in the first octant and is bounded by  $xz$ -coordinate plane, the  $yz$ -coordinate plane, above by the paraboloid  $z = 2 - x^2 - y^2$  and below by the plane  $z = 1$ .

A. Set up an iterated integral for finding the volume of  $R$ , where the first integration is with respect to  $y$ . Evaluate the integral by hand computation.

B. Set up an iterated integral for finding the volume of  $R$ , where the first integration is with respect to  $z$ . Evaluate the integral by hand computation.

Hint: Change the integral with respect to  $dx dy$  to polar coordinates.

---

40 pts. **Problem 3.** Let  $D$  be the solid bounded below by the cone  $z = 2\sqrt{x^2 + y^2}$  and above by the plane  $z = 4$ . Find  $\bar{z}$ , the  $z$ -coordinate of the centroid of  $D$ .

---

40 pts. **Problem 4.** Let  $D$  be the solid region inside the sphere  $x^2 + y^2 + z^2 = a^2$ . Calculate the integral

$$\iiint_D x^2 z^2 dV.$$

---

40 pts. **Problem 5.** Let the **surface**  $S$  be the upper hemisphere of the sphere  $x^2 + y^2 + z^2 = a^2$  (i.e., the part of the sphere in the region  $z \geq 0$ ). Find  $\bar{z}$ , the  $z$ -coordinate of the centroid of  $S$  and the moment of inertia of  $S$  for rotation about the  $z$ -axis. These are **surface integrals**, don't do the triple integral over a solid region.

---

40 pts. **Problem 6.** Let the surface  $S$  be the part of the cone  $z = \sqrt{x^2 + y^2}$  that lies in the region  $0 \leq z \leq 2$ . Let  $\mathbf{F}(x, y, z)$  be the vector field

$$\mathbf{F}(x, y, z) = xz\mathbf{i} + yz\mathbf{j} + z\mathbf{k}.$$

Calculate the flux integral

$$\iint_S \mathbf{F} \cdot \mathbf{n} dS,$$

where  $\mathbf{n}$  is the outward pointing unit normal on  $S$ .

---

40 pts.

**Problem 7.** Let the curve  $C$  be the part of the circle  $x^2 + y^2 = a^2$  that lies above the  $x$ -axis. Calculate the line integral

$$\int_C y \, ds.$$

Find  $\bar{y}$ , the  $y$ -coordinate of the centeroid of  $C$ .

---

40 pts.

**Problem 8.** Let  $C$  be the curve parametrized by

$$\mathbf{R}(t) = \frac{t^3}{3}\mathbf{i} + \frac{t^2}{2}\mathbf{j}, \quad 0 \leq t \leq 1.$$

Let the vectorfield  $\mathbf{F}(x, y)$  be defined by

$$\mathbf{F}(x, y) = xy\mathbf{i} + x^2\mathbf{j}.$$

Compute the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{R}.$$

---