
EXAM

Exam 3 (Take-home)

Math 3354, Fall 2008

November 25, 2008

- This is a Take-home exam, due on Friday, Dec. 5.
- Write all of your answers on separate sheets of paper. You can keep the exam questions.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- Unless otherwise instructed, you can use a calculator to do integrals and partial fractions decompositions. State clearly what you are going to put into the calculator and what you got out.
- You can use the textbook. You can discuss the problems with other people, but write up your own answers, don't just copy from someone else.
- This exam has 8 problems. There are **360 points total**.

Good luck!

40 pts. **Problem 1.** Use the method of variation of parameters to find the general solution of the Euler-Cauchy differential equation

$$t^2 x'' + 2tx' - 2x = t^2 \ln(t), \quad t > 0.$$

Caution: you have to rewrite the equation in standard form before applying the method of variation of parameters.

80 pts. **Problem 2.** In each part, find the inverse Laplace Transform. Unless otherwise stated you can use a calculator to find the Partial Fractions Decomposition.

A. In this part, find the partial fractions decomposition by hand.

$$\frac{4s^3 - 8s^2 + 2s - 3}{s^2(s^2 + 1)}$$

B.

$$\frac{2s - 5}{s^2 - 6s + 13}$$

C.

$$\frac{s^6 - 4s^5 + 6s^4 - 7s^3 - 10s^2 + 16s - 6}{s^4(s - 1)^3}$$

D.

$$\frac{s^3 + s^2 + 12s - 20}{(s^2 + 4)^2}$$

40 pts. **Problem 3.** In each part, use Laplace Transforms to solve the initial value problem.

A. $x'' - 3x' + 2x = te^{2t}$, $x(0) = 1$, $x'(0) = 2$.

B. $x'' + 4x = \sin(2t)$, $x(0) = 1$, $x'(0) = 1$.

40 pts. **Problem 4.** Find the Laplace Transform of the function

$$f(t) = \begin{cases} t, & 0 < t < 1, \\ t^2, & 1 < t < 2, \\ 0, & 2 < t < \pi, \\ \sin(t), & \pi < t < \infty. \end{cases}$$

40 pts. **Problem 5.** Find the inverse Laplace Transform of the function

$$F(s) = e^{-s} \left[\frac{1}{s} + \frac{2}{s^2} + \frac{4}{s^3} \right] + \frac{e^{-2s}}{(s+3)^2}.$$

40 pts. **Problem 6.** Use Laplace Transforms to find the solution of the following initial value problem.

$$x'' + 2x' + x = H(t-1) - H(t-2), \quad x(0) = 1, \quad x'(0) = 0.$$

Here $H(t)$ is the Heaviside function, of course.

Include a graph of $x(t)$ on the interval $0 < t < 5$. (Use Maple or some similar software.)

40 pts. **Problem 7.** Find the convolution $e^{2t} * t$ directly from the definition of convolution. Find the integral by hand computation.

40 pts. **Problem 8.** Find the convolution $e^{2t} * \sin(3t)$ using Laplace Transforms. (You can use a calculator to do the partial fractions.)
