

40 pts.

Problem 1. Find the general solution, or solve the initial value problem.

A.

$$\frac{dx}{dt} = -3t^2x^2.$$

B.

$$\frac{dx}{dt} = \frac{x}{1+2t}, \quad x(0) = 2.$$

40 pts.

Problem 2. Find the solution of the homogeneous equation

$$\frac{dx}{dt} + 2x = 0.$$

Then find the solutions of the following initial value problems, **using the method of undetermined coefficients** to find the particular solution.

A.

$$\frac{dx}{dt} + 2x = t^2 + 1, \quad x(0) = 1.$$

B.

$$\frac{dx}{dt} + 2x = e^t, \quad x(0) = 0$$

40 pts.

Problem 3. Find the general solution of the differential equation.

A.

$$\frac{dx}{dt} + \frac{3}{t}x = t.$$

B.

$$\frac{dx}{dt} + 2tx = t^3e^{-t^2}.$$

40 pts.

Problem 4. A cup of coffee at a temperature of 160 degrees (F) is brought into a room that has a temperature of 70 degrees. After 5 minutes the coffee has cooled to 140 degrees.

Find the formula for the temperature of the coffee as a function of time. At what time will the temperature of the coffee be 90 degrees?

40 pts.

Problem 5. A 100 gallon tank initially contains a brine solution with 70 pounds of salt dissolved in it. A brine solution with $1/2$ pound of salt per gallon enters the tank at a rate of 4 gallons per minute. A valve allows brine to flow out of the tank at a rate of 4 gallons per minute. The tank is kept well stirred.

Find the amount of salt (in pounds) in the tank as a function of time. At what time will there be 51 pounds of salt in the tank?

40 pts.

Problem 6. A very large tank initially contains 10 gallons of brine, which contains 5 pounds of salt. A brine solution containing 1 pound of salt per gallon enters the tank at 3 gallons per minute. A valve allows 1 gallon of brine per minute to leave the tank.

Find the formula for the amount of salt (in pounds) in the tank as a function of time. (Assume the tank does not fill up.)

EXAM

Exam 1

Math 3354, Fall 2008

October 8, 2008

- Write all of your answers on separate sheets of paper. You can keep the exam questions when you leave. You may leave when finished.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This exam has 6 problems. There are **240 points total**.

Good luck!