
EXAM

Exam 3, Take-home Exam

Math 2350, Sections 05 and 014, Fall 2008

Friday, November 28, 2008

- This is a Take-home exam, due on Friday, Dec. 5.
- Write all of your answers on separate sheets of paper. You can keep the exam questions.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- Unless otherwise instructed, you can use a calculator to do integrals. State clearly what you are going to put into the calculator and what you got out.
- You can use the textbook. You can discuss the problems with other people, but write up your own answers, don't just copy from someone else.
- This exam has 9 problems. There are **370 points total**.

Good luck!

40 pts. **Problem 1.** Let D be the solid in the first octant bounded by the coordinate planes and the sphere $x^2+y^2+z^2 = a^2$. Use spherical co-ordinates to find the integral

$$\iiint_D z^2 dV.$$

40 pts. **Problem 2.** Let D be the solid bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the upper hemisphere of the sphere $x^2+y^2+z^2 = 2$. Use cylindrical coordinates to find the moment of inertia of D for rotation around the z -axis. Assume the density is $\delta = 1$.

40 pts. **Problem 3.** Let the **surface** S be the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the xy -plane. Find the moment of inertia of S for rotation around the z -axis. Assume the density is $\delta = 1$.

Warning: This is a surface integral, not a triple integral.

40 pts. **Problem 4.** Let the surface S be the spherical surface $x^2 + y^2 + z^2 = a^2$. Let \mathbf{n} be the outward pointing unit normal field on S . Let $\mathbf{F}(x, y, z)$ be the vector field defined by

$$\mathbf{F}(x, y, z) = -yz\mathbf{i} + xz\mathbf{j} + z\mathbf{k}.$$

Calculate the Flux integral

$$\iint_S \mathbf{F} \cdot \mathbf{n} dS.$$

Warning: This is a surface integral, not a triple integral.

40 pts. **Problem 5.** Let the curve C be the part of the circle $x^2 + y^2 = a^2$ that lies above the x -axis. Calculate the line integral

$$\int_C y ds$$

and use it to find \bar{y} , the x -coordinate of the centroid of C .

Warning: This is a line integral, not a double integral.

40 pts. **Problem 6.** Let R be the region in the xy -plane bounded by the line $y = 0$ and the upper semicircle of the circle $x^2 + y^2 = a^2$.

A. Calculate the line integral

$$\int_{\partial R} y^2 dx + x dy.$$

B. Verify Green's theorem in this case computing the appropriate by double integral over R .

40 pts. **Problem 7.** Let the surface S be the upper hemisphere of the spherical surface $x^2 + y^2 + z^2 = 1$. Let \mathbf{n} be the inward pointing unit normal field on S (i.e., \mathbf{n} has a negative \mathbf{k} component). Let the vector field \mathbf{F} be given by

$$\mathbf{F}(x, y, z) = -y\mathbf{i} + x\mathbf{j} + z^2\mathbf{k}.$$

Verify Stoke's theorem

$$\int_{\partial S} \mathbf{F} \cdot d\mathbf{R} = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} dS.$$

in this case.

40 pts. **Problem 8.** Let D be the solid bounded by the sphere $x^2 + y^2 + z^2 = a^2$. Let \mathbf{n} be the outward pointing unit normal field on ∂D . Let the vector field \mathbf{F} be given by

$$\mathbf{F}(x, y, z) = (x^3 + z)\mathbf{i} + (y^3 + x)\mathbf{j} + (z^3 + x + y)\mathbf{k}.$$

Use the Divergence Theorem to find the flux

$$\iint_{\partial S} \mathbf{F} \cdot \mathbf{n} dS$$

by calculating a triple integral instead.

50 pts.

Problem 9. Let D be the solid that is bounded below by the xy -plane and above by the upper hemisphere of the spherical surface $x^2 + y^2 + z^2 = a^2$. Let \mathbf{F} be the vector field given by

$$\mathbf{F}(x, y, z) = xz\mathbf{i} + yz\mathbf{j} + (x^2 + y^2)\mathbf{k}.$$

Let the surface S_1 be the upper hemisphere of the spherical surface $x^2 + y^2 + z^2 = a^2$. Let the surface S_2 be the circular disk in the xy -plane bounded by the circle $x^2 + y^2 = a^2$. Thus, $\partial D = S_1 + S_2$ and the Divergence Theorem tells us

$$(*) \quad \iiint_D \nabla \cdot \mathbf{F} \, dV = \iint_{\partial D} \mathbf{F} \cdot \mathbf{n} \, dS = \iint_{S_1} \mathbf{F} \cdot \mathbf{n} \, dS + \iint_{S_2} \mathbf{F} \cdot \mathbf{n} \, dS$$

A. Compute the integral

$$\iiint_D \nabla \cdot \mathbf{F} \, dV.$$

B. Compute the integral

$$\iint_{S_2} \mathbf{F} \cdot \mathbf{n} \, dS$$

Which normal \mathbf{n} should you use?

C. Now use equation (*) to find

$$\iint_{S_1} \mathbf{F} \cdot \mathbf{n} \, dS$$

without having to do the integration.
