

50 pts.

Problem 1. In each part you are given the augmented matrix of a system of linear equations, with the coefficient matrix in reduced row echelon form. Determine if the system is consistent and, if it is consistent, find all solutions.

A.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right]$$

B.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

C.

$$\left[\begin{array}{cccccc|c} 1 & -2 & 3 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 & 1 & 3 & -5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

45 pts.

Problem 2. Consider row operations on matrices with 3 rows.

Recall that for each row operation there is a corresponding elementary matrix E so that EA is the same as the matrix obtained by applying the row operation to A .

A. Consider the row operation $R_1 \leftrightarrow R_3$.

- i.) Find the corresponding elementary matrix E .
- ii.) Find the inverse row operation.
- iii.) Find the elementary matrix that corresponds to the inverse row operation.

B. Consider the row operation $R_2 \leftarrow 5R_2$.

- i.) Find the corresponding elementary matrix E .
- ii.) Find the inverse row operation.
- iii.) Find the elementary matrix that corresponds to the inverse row operation.

C. Consider the row operation $R_3 \leftarrow R_3 - 3R_1$

- i.) Find the corresponding elementary matrix E .
- ii.) Find the inverse row operation.
- iii.) Find the elementary matrix that corresponds to the inverse row operation.

40 pts.

Problem 3. In each part, use row operations to determine if the matrix A is invertible and, if so, to find the inverse. It is not necessary to show the individual row operations (you can just use the `rref` key on the calculator). Show the augmented matrix you start with and the augmented matrix you finish with. Give the matrix entries in fractional form.

A.

$$\begin{bmatrix} 4 & 3 & 1 \\ 4 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

B.

$$\begin{bmatrix} 0 & 1 & 5 \\ 1 & 1 & 3 \\ 1 & 2 & 8 \end{bmatrix}$$

60 pts.

Problem 4. The matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 5 \\ 0 & 1 & 0 \end{bmatrix}$$

is invertible. Use row operations to express A as a product of elementary matrices. You can use a calculator to do the row operations, but you'll have to show each row operation.

70 pts.

Problem 5. Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 5 & 1 \\ 1 & 5 & 1 \end{bmatrix}$$

- A. Find the cofactors A_{12} and A_{33} .
 - B. Compute $\det(A)$, using the cofactor expansion along a selected row or column.
 - C. Find the adjoint matrix $\text{adj}(A)$ from its definition in terms of cofactors.
 - D. Use the information above to find A^{-1} .
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40 pts.

Problem 6. Find the following determinant by the method of elimination, i.e., by using row operations and keeping track of the effect of the row operations on the determinant. Sorry, no credit for finding it by another method.

$$\begin{vmatrix} 3 & 17 & 2 \\ 2 & 6 & 2 \\ 1 & -1 & -1 \end{vmatrix}$$

40 pts.

Problem 7. Use Cramer's rule to solve the following system.

$$\begin{aligned} 2x + 2y &= 5 \\ 3x + 5y &= -1 \end{aligned}$$

EXAM

Exam #1

Math 2360, Summer 2007

July 23, 2007

- Write all of your answers on separate sheets of paper. You can keep the exam questions when you leave. You may leave when finished.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This exam has 7 problems. There are **345 points total**.

Good luck!