
EXAM

Exam 3
Take Home Exam

Math 3322, Spring 2007

April 25, 2007

- This is a takehome exam **due May 3**.
- You can look at the book and your notes. You may discuss the exam with other students, but write up your own solutions!
- Write all of your answers on separate sheets of paper. You can keep the exam questions.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This exam has 7 problems. There are **340 points total**.

Good luck!

40 pts. **Problem 1.** A 200 gallon tank contains 150 pounds of dissolved salt. Saltwater containing $1/2$ pound of salt per gallon is poured into the tank at a rate of 4 gallons per minute. Saltwater flows out of the tank at 4 gallons per minute. Assume that the tank is well stirred, so the concentration of salt is uniform.

Find an expression for the amount of y of salt in the tank as a function of time. At what time will there be 101 pounds of salt in the tank?

60 pts. **Problem 2.** In each part, use the method of undetermined coefficients to find the general solution of the differential equation.

A. $(D^2 + 3D + 2)y = x^2 + 1$

B. $(D^2 + 3D + 2)y = xe^{-2x}$.

C. $(D^2 + 1)y = \cos(t)$.

40 pts. **Problem 3.** In each part, find the Laplace transform. Use the table on page 453 of the text.

A. $f(t) = 3t^4 - 2t^3 + t^2 - 2t + 1$.

B. $f(t) = t^3e^{4t} + t^2e^{-3t}$.

C. $f(t) = t \cos(2t)$.

D. $f(t) = 3e^{-2t} \cos(3t) + 4e^{-2t} \sin(3t)$.

60 pts. **Problem 4.** In each part, find the form of the partial fraction decomposition. In other words, write out the expression for the partial fraction decomposition with undetermined coefficients, but **do not** solve for the coefficients. If you're doing any computations, you're doing more than I'm asking for.

A.
$$\frac{s^2 + 1}{(s - 1)(s + 1)(s - 2)}$$

B.
$$\frac{1}{s^3(s^2 + 1)}$$

C.
$$\frac{1}{s(s^2 + 1)^2}$$

40 pts. **Problem 5.** In each part, find the partial fraction decomposition of the given rational function **using hand computation**. Show your work!

A.
$$\frac{2s^2 - s + 2}{s^2(s - 1)}.$$

B.
$$\frac{s^2 - s + 1}{s(s^2 + 1)}$$

60 pts. **Problem 6.** In each part, find the inverse Laplace transform of the given function. Use a calculator for the partial fractions calculations.

A.
$$F(s) = \frac{s^4 + s^3 + s^2 + 11s + 18}{s^4(s + 2)}$$

B.
$$F(s) = \frac{3s^2 - 11s + 60}{(s^2 - 4s + 29)(s - 1)}$$

C.
$$F(s) = \frac{s^2 + 4s - 4}{(s^2 + 1)^2}$$

40 pts. **Problem 7.** Use **Laplace transforms** to solve the differential equations. You can use a calculator to do the partial fraction decompositions.

A. $y'' - 3y' + 2y = e^{2t}$, $y(0) = 1$, $y'(0) = 1$.

B. $y'' + 4y = \cos(2t)$, $y(0) = 1$, $y'(0) = 0$.
