

# Using the Jordan Decomposition to Find the Matrix Exponential

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> restart;
> with(LinearAlgebra):
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>
```

Consider the system  $x' = Ax$ , where  $A$  is the matrix

```
> A := Matrix([[22, -12, -23, -13, -31], [19, -10, -21, -12, -28]
, [58, -83, 21, -1, 71], [-54, 82, -25, 0, -80], [-14, 24, -12,
-3, -29]]);
```

$$A := \begin{bmatrix} 22 & -12 & -23 & -13 & -31 \\ 19 & -10 & -21 & -12 & -28 \\ 58 & -83 & 21 & -1 & 71 \\ -54 & 82 & -25 & 0 & -80 \\ -14 & 24 & -12 & -3 & -29 \end{bmatrix}$$

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First, we find the eigenvalues and eigenvectors of  $A$ .

```
> Eigenvectors(A);
```

$$\begin{bmatrix} 2 \\ 2 \\ 2 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 & 0 & 0 & 1 & 0 \\ -2 & 0 & 0 & 1 & 0 \\ -6 & 0 & 0 & -2 & 0 \\ 7 & 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

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The eigenspace for eigenvalue 2 is one dimensional and the eigenspace for eigenvalue -1 is one dimensional. The matrix is not diagonalizable.

The generalized eigenspace for 2 is the nullspace of

```
> B1 := (A-2)^5;
```

$$B1 := \begin{bmatrix} -2025 & 3078 & -1053 & -162 & -3078 \\ -2025 & 3078 & -1053 & -162 & -3078 \\ 12069 & -15633 & 1377 & -1134 & 9072 \\ -12069 & 15633 & -1377 & 1134 & -9072 \\ -4698 & 6237 & -810 & 324 & -4050 \end{bmatrix}$$

```
> bg1 := NullSpace(B1);
```

$$bg1 := \left\{ \begin{bmatrix} \frac{114}{31} \\ \frac{106}{31} \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} \frac{34}{31} \\ \frac{24}{31} \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{69}{31} \\ \frac{56}{31} \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$$

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For eigenvalue -1, the generalized eigenspace is the nullspace of

> **B2 := (A+1)^5;**

$$B2 := \begin{bmatrix} 2538 & -1377 & -2808 & -1647 & -3483 \\ 1890 & -1134 & -1998 & -1242 & -2268 \\ 324 & -2673 & 3240 & 891 & 7047 \\ 1566 & 2268 & -5967 & -2133 & -11502 \\ 27 & 567 & -945 & -351 & -1782 \end{bmatrix}$$

> **bg2 := NullSpace(B2);**

$$bg2 := \left\{ \begin{bmatrix} 3 \\ 3 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

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Put these together in a matrix  $P$ .

> **bg1 := convert(bg1, list);**

$$bg1 := \left( \begin{bmatrix} \frac{69}{31} \\ \frac{56}{31} \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{34}{31} \\ \frac{24}{31} \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{114}{31} \\ \frac{106}{31} \\ 0 \\ 0 \\ 1 \end{bmatrix} \right)$$

> **bg2 := convert(bg2, list);**

$$bg2 := \left[ \begin{array}{c} \left[ \begin{array}{c} -1 \\ -1 \\ -1 \\ 1 \\ 0 \end{array} \right], \left[ \begin{array}{c} 3 \\ 3 \\ 0 \\ 0 \\ 1 \end{array} \right] \end{array} \right]$$

> `P := <Matrix(bg1) | Matrix(bg2)>;`

$$P := \left[ \begin{array}{ccccc} \frac{69}{31} & \frac{34}{31} & \frac{114}{31} & -1 & 3 \\ \frac{56}{31} & \frac{24}{31} & \frac{106}{31} & -1 & 3 \\ 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{array} \right]$$

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> `d := DiagonalMatrix([2, 2, 2, -1, -1]);`

$$d := \left[ \begin{array}{ccccc} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{array} \right]$$

>

The diagonalizable part  $S$  of  $A$  satisfies  $SP = Pd$ , so

> `S := P.d.P^(-1);`

$$S := \left[ \begin{array}{ccccc} 32 & -27 & -18 & -12 & -18 \\ 30 & -25 & -18 & -12 & -18 \\ 39 & -63 & 29 & 6 & 72 \\ -39 & 63 & -27 & -4 & -72 \\ -3 & 12 & -15 & -6 & -28 \end{array} \right]$$

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Check that  $S$  commutes with  $A$ .

> `A.S-S.A;`

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

>  $N := A - S;$

$$N := \begin{bmatrix} -10 & 15 & -5 & -1 & -13 \\ -11 & 15 & -3 & 0 & -10 \\ 19 & -20 & -8 & -7 & -1 \\ -15 & 19 & 2 & 4 & -8 \\ -11 & 12 & 3 & 3 & -1 \end{bmatrix}$$

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Check that  $N$  is nilpotent.

>  $N^5;$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

>  $N^2; N^3; N^4;$

$$\begin{bmatrix} -2 & 0 & 4 & 2 & 6 \\ -2 & 0 & 4 & 2 & 6 \\ -6 & 0 & 12 & 6 & 18 \\ 7 & 0 & -14 & -7 & -21 \\ 1 & 0 & -2 & -1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

>

Thus,  $A = S + N$  is the Jordan Decomposition of  $A$ .

Now find  $e^{At}$ . Since  $S$  and  $N$  commute,  $e^{tA} = e^{tS + tN} = e^{tS} e^{tN}$ .

Since  $S = PdP^{-1}$ ,  $e^{tS} = Pe^{td}P^{-1}$ , so

> `expd := DiagonalMatrix([exp(2*t), exp(2*t), exp(2*t), exp(-t), exp(-t)]);`

$$\text{expd} := \begin{bmatrix} e^{2t} & 0 & 0 & 0 & 0 \\ 0 & e^{2t} & 0 & 0 & 0 \\ 0 & 0 & e^{2t} & 0 & 0 \\ 0 & 0 & 0 & e^{-t} & 0 \\ 0 & 0 & 0 & 0 & e^{-t} \end{bmatrix}$$

> `exps := P.expd.P^(-1);`

$$\text{exps} := \begin{bmatrix} 11e^{2t} - 10e^{-t} & -9e^{2t} + 9e^{-t} & -6e^{2t} + 6e^{-t} & -4e^{2t} + 4e^{-t} & -6e^{2t} + 6e^{-t} \\ 10e^{2t} - 10e^{-t} & -8e^{2t} + 9e^{-t} & -6e^{2t} + 6e^{-t} & -4e^{2t} + 4e^{-t} & -6e^{2t} + 6e^{-t} \\ 13e^{2t} - 13e^{-t} & -21e^{2t} + 21e^{-t} & 10e^{2t} - 9e^{-t} & 2e^{2t} - 2e^{-t} & 24e^{2t} - 24e^{-t} \\ -13e^{2t} + 13e^{-t} & 21e^{2t} - 21e^{-t} & -9e^{2t} + 9e^{-t} & -e^{2t} + 2e^{-t} & -24e^{2t} + 24e^{-t} \\ -e^{2t} + e^{-t} & 4e^{2t} - 4e^{-t} & -5e^{2t} + 5e^{-t} & -2e^{2t} + 2e^{-t} & -9e^{2t} + 10e^{-t} \end{bmatrix}$$

>

For  $e^{tN}$ , use the power series, remembering that  $N^3 = 0$ .

> `expn := IdentityMatrix(5)+t*N + (t*N)^2/2;`

$$\text{expn} := \begin{bmatrix} 1 - 10t - t^2 & 15t & -5t + 2t^2 & -t + t^2 & -13t + 3t^2 \\ -11t - t^2 & 1 + 15t & -3t + 2t^2 & t^2 & -10t + 3t^2 \\ 19t - 3t^2 & -20t & 1 - 8t + 6t^2 & -7t + 3t^2 & -t + 9t^2 \\ -15t + \frac{7}{2}t^2 & 19t & 2t - 7t^2 & 1 + 4t - \frac{7}{2}t^2 & -8t - \frac{21}{2}t^2 \\ -11t + \frac{1}{2}t^2 & 12t & 3t - t^2 & 3t - \frac{1}{2}t^2 & 1 - t - \frac{3}{2}t^2 \end{bmatrix}$$

>

Thus,

> **expa := simplify(exps.expn);**

$$\begin{aligned} \text{expa} := & \left[ \left[ 11 e^{2t} + e^{2t} t - e^{2t} t^2 - 10 e^{-t} - 11 e^{-t} t, 2 e^{2t} t + 13 e^{-t} t - 9 e^{2t} + 9 e^{-t}, -6 e^{2t} t + 2 e^{2t} t^2 \right. \right. \\ & + e^{-t} t - 6 e^{2t} + 6 e^{-t}, -3 e^{2t} t + e^{2t} t^2 + 2 e^{-t} t - 4 e^{2t} + 4 e^{-t}, -9 e^{2t} t + 3 e^{2t} t^2 - 4 e^{-t} t - 6 e^{2t} \\ & \left. \left. + 6 e^{-t} \right], \right. \\ & \left[ 10 e^{2t} - e^{2t} t^2 - 10 e^{-t} - 11 e^{-t} t, 2 e^{2t} t + 13 e^{-t} t - 8 e^{2t} + 9 e^{-t}, -4 e^{2t} t + 2 e^{2t} t^2 + e^{-t} t \right. \\ & \left. - 6 e^{2t} + 6 e^{-t}, -2 e^{2t} t + e^{2t} t^2 + 2 e^{-t} t - 4 e^{2t} + 4 e^{-t}, -6 e^{2t} t + 3 e^{2t} t^2 - 4 e^{-t} t - 6 e^{2t} + 6 e^{-t} \right. \\ & \left. \right], \\ & \left[ 13 e^{2t} - 3 e^{2t} t - 3 e^{2t} t^2 - 13 e^{-t} + 22 e^{-t} t, 6 e^{2t} t - 26 e^{-t} t - 21 e^{2t} + 21 e^{-t}, -6 e^{2t} t + 6 e^{2t} t^2 \right. \\ & \left. - 2 e^{-t} t + 10 e^{2t} - 9 e^{-t}, -3 e^{2t} t + 3 e^{2t} t^2 - 4 e^{-t} t + 2 e^{2t} - 2 e^{-t}, -9 e^{2t} t + 9 e^{2t} t^2 + 8 e^{-t} t \right. \\ & \left. + 24 e^{2t} - 24 e^{-t} \right], \\ & \left[ -13 e^{2t} + 7 e^{2t} t + \frac{7 e^{2t} t^2}{2} + 13 e^{-t} - 22 e^{-t} t, -7 e^{2t} t + 26 e^{-t} t + 21 e^{2t} - 21 e^{-t}, -7 e^{2t} t^2 \right. \\ & \left. + 2 e^{-t} t - 9 e^{2t} + 9 e^{-t}, -\frac{7 e^{2t} t^2}{2} + 4 e^{-t} t - e^{2t} + 2 e^{-t}, -\frac{21 e^{2t} t^2}{2} - 8 e^{-t} t - 24 e^{2t} + 24 e^{-t} \right], \\ & \left[ -e^{2t} + \frac{e^{2t} t^2}{2} + e^{-t} - 11 e^{-t} t, -e^{2t} t + 13 e^{-t} t + 4 e^{2t} - 4 e^{-t}, 2 e^{2t} t - e^{2t} t^2 + e^{-t} t - 5 e^{2t} \right. \\ & \left. + 5 e^{-t}, e^{2t} t - \frac{e^{2t} t^2}{2} + 2 e^{-t} t - 2 e^{2t} + 2 e^{-t}, 3 e^{2t} t - \frac{3 e^{2t} t^2}{2} - 4 e^{-t} t - 9 e^{2t} + 10 e^{-t} \right] \end{aligned}$$

> **map(collect, expa, [exp(2\*t),exp(-t)]);**

$$\begin{aligned} & \left[ \left( (t - t^2 + 11) e^{2t} + (-10 - 11t) e^{-t}, (2t - 9) e^{2t} + (13t + 9) e^{-t}, (2t^2 - 6 - 6t) e^{2t} + (6 \right. \right. \\ & \left. \left. + t) e^{-t}, (t^2 - 4 - 3t) e^{2t} + (4 + 2t) e^{-t}, (3t^2 - 6 - 9t) e^{2t} + (6 - 4t) e^{-t} \right), \right. \\ & \left[ (10 - t^2) e^{2t} + (-10 - 11t) e^{-t}, (2t - 8) e^{2t} + (13t + 9) e^{-t}, (2t^2 - 6 - 4t) e^{2t} + (6 \right. \\ & \left. + t) e^{-t}, (t^2 - 4 - 2t) e^{2t} + (4 + 2t) e^{-t}, (3t^2 - 6 - 6t) e^{2t} + (6 - 4t) e^{-t} \right], \\ & \left[ (-3t - 3t^2 + 13) e^{2t} + (22t - 13) e^{-t}, (6t - 21) e^{2t} + (-26t + 21) e^{-t}, (6t^2 + 10 - 6t) e^{2t} \right. \\ & \left. + (-9 - 2t) e^{-t}, (3t^2 + 2 - 3t) e^{2t} + (-2 - 4t) e^{-t}, (9t^2 + 24 - 9t) e^{2t} + (-24 + 8t) e^{-t} \right], \\ & \left[ \left( 7t + \frac{7}{2} t^2 - 13 \right) e^{2t} + (-22t + 13) e^{-t}, (-7t + 21) e^{2t} + (26t - 21) e^{-t}, (-7t^2 - 9) e^{2t} \right. \\ & \left. + (2t + 9) e^{-t}, \left( -\frac{7t^2}{2} - 1 \right) e^{2t} + (4t + 2) e^{-t}, \left( -\frac{21t^2}{2} - 24 \right) e^{2t} + (-8t + 24) e^{-t} \right], \\ & \left[ \left( -1 + \frac{t^2}{2} \right) e^{2t} + (1 - 11t) e^{-t}, (-t + 4) e^{2t} + (13t - 4) e^{-t}, (-t^2 - 5 + 2t) e^{2t} + (5 \right. \\ & \left. + t) e^{-t}, \left( -\frac{1}{2} t^2 - 2 + t \right) e^{2t} + (2 + 2t) e^{-t}, \left( -\frac{3}{2} t^2 - 9 + 3t \right) e^{2t} + (10 - 4t) e^{-t} \right] \end{aligned}$$

>

Let  $x_0$  be the initial condition for the solution of  $x' = Ax$ .

> **x0 := Vector(5, symbol=c);**

$$x0 := \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix}$$

>

The solution then is

> **xsol := expa.x0;**

$$\begin{aligned} xsol := & \left[ \left[ (11 e^{2t} + e^{2t} t - e^{2t} t^2 - 10 e^{-t} - 11 e^{-t} t) c_1 + (2 e^{2t} t + 13 e^{-t} t - 9 e^{2t} + 9 e^{-t}) c_2 + \right. \right. \\ & \left. \left. -6 e^{2t} t + 2 e^{2t} t^2 + e^{-t} t - 6 e^{2t} + 6 e^{-t}) c_3 + (-3 e^{2t} t + e^{2t} t^2 + 2 e^{-t} t - 4 e^{2t} + 4 e^{-t}) c_4 + \right. \right. \\ & \left. \left. -9 e^{2t} t + 3 e^{2t} t^2 - 4 e^{-t} t - 6 e^{2t} + 6 e^{-t}) c_5 \right], \right. \\ & \left[ (10 e^{2t} - e^{2t} t^2 - 10 e^{-t} - 11 e^{-t} t) c_1 + (2 e^{2t} t + 13 e^{-t} t - 8 e^{2t} + 9 e^{-t}) c_2 + (-4 e^{2t} t \right. \\ & \left. + 2 e^{2t} t^2 + e^{-t} t - 6 e^{2t} + 6 e^{-t}) c_3 + (-2 e^{2t} t + e^{2t} t^2 + 2 e^{-t} t - 4 e^{2t} + 4 e^{-t}) c_4 + (-6 e^{2t} t \right. \\ & \left. + 3 e^{2t} t^2 - 4 e^{-t} t - 6 e^{2t} + 6 e^{-t}) c_5 \right], \\ & \left[ (13 e^{2t} - 3 e^{2t} t - 3 e^{2t} t^2 - 13 e^{-t} + 22 e^{-t} t) c_1 + (6 e^{2t} t - 26 e^{-t} t - 21 e^{2t} + 21 e^{-t}) c_2 + \right. \\ & \left. -6 e^{2t} t + 6 e^{2t} t^2 - 2 e^{-t} t + 10 e^{2t} - 9 e^{-t}) c_3 + (-3 e^{2t} t + 3 e^{2t} t^2 - 4 e^{-t} t + 2 e^{2t} - 2 e^{-t}) c_4 \right. \\ & \left. + (-9 e^{2t} t + 9 e^{2t} t^2 + 8 e^{-t} t + 24 e^{2t} - 24 e^{-t}) c_5 \right], \\ & \left[ \left( -13 e^{2t} + 7 e^{2t} t + \frac{7 e^{2t} t^2}{2} + 13 e^{-t} - 22 e^{-t} t \right) c_1 + (-7 e^{2t} t + 26 e^{-t} t + 21 e^{2t} - 21 e^{-t}) c_2 \right. \\ & \left. + (-7 e^{2t} t^2 + 2 e^{-t} t - 9 e^{2t} + 9 e^{-t}) c_3 + \left( -\frac{7 e^{2t} t^2}{2} + 4 e^{-t} t - e^{2t} + 2 e^{-t} \right) c_4 + \left( -\frac{21 e^{2t} t^2}{2} \right. \right. \\ & \left. \left. - 8 e^{-t} t - 24 e^{2t} + 24 e^{-t} \right) c_5 \right], \\ & \left[ \left( -e^{2t} + \frac{e^{2t} t^2}{2} + e^{-t} - 11 e^{-t} t \right) c_1 + (-e^{2t} t + 13 e^{-t} t + 4 e^{2t} - 4 e^{-t}) c_2 + (2 e^{2t} t - e^{2t} t^2 \right. \right. \\ & \left. \left. + e^{-t} t - 5 e^{2t} + 5 e^{-t}) c_3 + \left( e^{2t} t - \frac{e^{2t} t^2}{2} + 2 e^{-t} t - 2 e^{2t} + 2 e^{-t} \right) c_4 + \left( 3 e^{2t} t - \frac{3 e^{2t} t^2}{2} \right. \right. \\ & \left. \left. - 4 e^{-t} t - 9 e^{2t} + 10 e^{-t} \right) c_5 \right] \end{aligned}$$

>

Let's check.

> **xsolprime := map(diff, xsol, t);**

$$\begin{aligned} xsolprime := & \left[ \left[ (23 e^{2t} - 2 e^{2t} t^2 - e^{-t} + 11 e^{-t} t) c_1 + (4 e^{2t} t - 16 e^{2t} - 13 e^{-t} t + 4 e^{-t}) c_2 + \right. \right. \\ & \left. \left. -8 e^{2t} t - 18 e^{2t} + 4 e^{2t} t^2 - e^{-t} t - 5 e^{-t}) c_3 + (-4 e^{2t} t - 11 e^{2t} + 2 e^{2t} t^2 - 2 e^{-t} t - 2 e^{-t}) c_4 \right. \right. \end{aligned}$$

$$\begin{aligned}
& + (-12 e^{2t} t - 21 e^{2t} + 6 e^{2t} t^2 + 4 e^{-t} t - 10 e^{-t}) c_5], \\
& [(20 e^{2t} - 2 e^{2t} t^2 - 2 e^{2t} t - e^{-t} + 11 e^{-t} t) c_1 + (4 e^{2t} t - 14 e^{2t} - 13 e^{-t} t + 4 e^{-t}) c_2 + (-4 e^{2t} t - 16 e^{2t} + 4 e^{2t} t^2 - e^{-t} t - 5 e^{-t}) c_3 + (-2 e^{2t} t - 10 e^{2t} + 2 e^{2t} t^2 - 2 e^{-t} t - 2 e^{-t}) c_4 \\
& + (-6 e^{2t} t - 18 e^{2t} + 6 e^{2t} t^2 + 4 e^{-t} t - 10 e^{-t}) c_5], \\
& [(23 e^{2t} - 12 e^{2t} t - 6 e^{2t} t^2 + 35 e^{-t} - 22 e^{-t} t) c_1 + (12 e^{2t} t - 36 e^{2t} + 26 e^{-t} t - 47 e^{-t}) c_2 \\
& + (14 e^{2t} + 12 e^{2t} t^2 + 2 e^{-t} t + 7 e^{-t}) c_3 + (e^{2t} + 6 e^{2t} t^2 + 4 e^{-t} t - 2 e^{-t}) c_4 + (39 e^{2t} \\
& + 18 e^{2t} t^2 - 8 e^{-t} t + 32 e^{-t}) c_5], \\
& [(-19 e^{2t} + 21 e^{2t} t + 7 e^{2t} t^2 - 35 e^{-t} + 22 e^{-t} t) c_1 + (-14 e^{2t} t + 35 e^{2t} - 26 e^{-t} t + 47 e^{-t}) c_2 \\
& + (-14 e^{2t} t^2 - 14 e^{2t} t - 2 e^{-t} t - 7 e^{-t} - 18 e^{2t}) c_3 + (-7 e^{2t} t^2 - 7 e^{2t} t - 4 e^{-t} t + 2 e^{-t} \\
& - 2 e^{2t}) c_4 + (-21 e^{2t} t^2 - 21 e^{2t} t + 8 e^{-t} t - 32 e^{-t} - 48 e^{2t}) c_5], \\
& [(-2 e^{2t} + e^{2t} t^2 + e^{2t} t - 12 e^{-t} + 11 e^{-t} t) c_1 + (-2 e^{2t} t + 7 e^{2t} - 13 e^{-t} t + 17 e^{-t}) c_2 \\
& + (2 e^{2t} t - 8 e^{2t} - 2 e^{2t} t^2 - e^{-t} t - 4 e^{-t}) c_3 + (e^{2t} t - 3 e^{2t} - e^{2t} t^2 - 2 e^{-t} t) c_4 + (3 e^{2t} t \\
& - 15 e^{2t} - 3 e^{2t} t^2 + 4 e^{-t} t - 14 e^{-t}) c_5]]
\end{aligned}$$

> **xsolprime - A.xsol;**

$$\begin{aligned}
& \left[ \left[ 12 (-4 e^{2t} t + 2 e^{2t} t^2 + e^{-t} t - 6 e^{2t} + 6 e^{-t}) c_3 + 12 (-2 e^{2t} t + e^{2t} t^2 + 2 e^{-t} t - 4 e^{2t} + 4 e^{-t}) c_4 \right. \right. \\
& + 12 (-6 e^{2t} t + 3 e^{2t} t^2 - 4 e^{-t} t - 6 e^{2t} + 6 e^{-t}) c_5 + 23 (13 e^{2t} - 3 e^{2t} t - 3 e^{2t} t^2 - 13 e^{-t} \\
& + 22 e^{-t} t) c_1 + 23 (6 e^{2t} t - 26 e^{-t} t - 21 e^{2t} + 21 e^{-t}) c_2 + (23 e^{2t} - 2 e^{2t} t^2 - e^{-t} \\
& + 11 e^{-t} t) c_1 + (4 e^{2t} t - 16 e^{2t} - 13 e^{-t} t + 4 e^{-t}) c_2 + (-8 e^{2t} t - 18 e^{2t} + 4 e^{2t} t^2 - e^{-t} t \\
& - 5 e^{-t}) c_3 + 23 (-6 e^{2t} t + 6 e^{2t} t^2 - 2 e^{-t} t + 10 e^{2t} - 9 e^{-t}) c_3 + 23 (-3 e^{2t} t + 3 e^{2t} t^2 \\
& - 4 e^{-t} t + 2 e^{2t} - 2 e^{-t}) c_4 + 23 (-9 e^{2t} t + 9 e^{2t} t^2 + 8 e^{-t} t + 24 e^{2t} - 24 e^{-t}) c_5 + 13 \left( -13 e^{2t} \right. \\
& + 7 e^{2t} t + \frac{7 e^{2t} t^2}{2} + 13 e^{-t} - 22 e^{-t} t \left. \right) c_1 + 13 (-7 e^{2t} t + 26 e^{-t} t + 21 e^{2t} - 21 e^{-t}) c_2 + 13 (- \\
& -7 e^{2t} t^2 + 2 e^{-t} t - 9 e^{2t} + 9 e^{-t}) c_3 + (-4 e^{2t} t - 11 e^{2t} + 2 e^{2t} t^2 - 2 e^{-t} t - 2 e^{-t}) c_4 + ( \\
& -12 e^{2t} t - 21 e^{2t} + 6 e^{2t} t^2 + 4 e^{-t} t - 10 e^{-t}) c_5 + 12 (2 e^{2t} t + 13 e^{-t} t - 8 e^{2t} + 9 e^{-t}) c_2 \\
& + 13 \left( -\frac{7 e^{2t} t^2}{2} + 4 e^{-t} t - e^{2t} + 2 e^{-t} \right) c_4 + 13 \left( -\frac{21 e^{2t} t^2}{2} - 8 e^{-t} t - 24 e^{2t} + 24 e^{-t} \right) c_5
\end{aligned}$$



$$\begin{aligned}
& + 31 \left( -e^{2t} + \frac{e^{2t} t^2}{2} + e^{-t} - 11 e^{-t} t \right) c_1 + 31 \left( -e^{2t} t + 13 e^{-t} t + 4 e^{2t} - 4 e^{-t} \right) c_2 + 31 \left( 2 e^{2t} t \right. \\
& - e^{2t} t^2 + e^{-t} t - 5 e^{2t} + 5 e^{-t} \left. \right) c_3 + 31 \left( e^{2t} t - \frac{e^{2t} t^2}{2} + 2 e^{-t} t - 2 e^{2t} + 2 e^{-t} \right) c_4 + 31 \left( 3 e^{2t} t \right. \\
& - \frac{3 e^{2t} t^2}{2} - 4 e^{-t} t - 9 e^{2t} + 10 e^{-t} \left. \right) c_5 - 22 \left( -9 e^{2t} t + 3 e^{2t} t^2 - 4 e^{-t} t - 6 e^{2t} + 6 e^{-t} \right) c_5 \\
& - 22 \left( -3 e^{2t} t + e^{2t} t^2 + 2 e^{-t} t - 4 e^{2t} + 4 e^{-t} \right) c_4 - 22 \left( -6 e^{2t} t + 2 e^{2t} t^2 + e^{-t} t - 6 e^{2t} \right. \\
& + 6 e^{-t} \left. \right) c_3 - 22 \left( 2 e^{2t} t + 13 e^{-t} t - 9 e^{2t} + 9 e^{-t} \right) c_2 - 22 \left( 11 e^{2t} + e^{2t} t - e^{2t} t^2 - 10 e^{-t} \right. \\
& - 11 e^{-t} t \left. \right) c_1 + 12 \left( 10 e^{2t} - e^{2t} t^2 - 10 e^{-t} - 11 e^{-t} t \right) c_1], \\
& \left[ 10 \left( -4 e^{2t} t + 2 e^{2t} t^2 + e^{-t} t - 6 e^{2t} + 6 e^{-t} \right) c_3 + 10 \left( -2 e^{2t} t + e^{2t} t^2 + 2 e^{-t} t - 4 e^{2t} \right. \right. \\
& + 4 e^{-t} \left. \right) c_4 + 10 \left( -6 e^{2t} t + 3 e^{2t} t^2 - 4 e^{-t} t - 6 e^{2t} + 6 e^{-t} \right) c_5 + 21 \left( 13 e^{2t} - 3 e^{2t} t - 3 e^{2t} t^2 \right. \\
& - 13 e^{-t} + 22 e^{-t} t \left. \right) c_1 + 21 \left( 6 e^{2t} t - 26 e^{-t} t - 21 e^{2t} + 21 e^{-t} \right) c_2 + \left( 20 e^{2t} - 2 e^{2t} t^2 - 2 e^{2t} t \right. \\
& - e^{-t} + 11 e^{-t} t \left. \right) c_1 + \left( 4 e^{2t} t - 14 e^{2t} - 13 e^{-t} t + 4 e^{-t} \right) c_2 + \left( -4 e^{2t} t - 16 e^{2t} + 4 e^{2t} t^2 - e^{-t} t \right. \\
& - 5 e^{-t} \left. \right) c_3 + \left( -2 e^{2t} t - 10 e^{2t} + 2 e^{2t} t^2 - 2 e^{-t} t - 2 e^{-t} \right) c_4 + \left( -6 e^{2t} t - 18 e^{2t} + 6 e^{2t} t^2 \right. \\
& + 4 e^{-t} t - 10 e^{-t} \left. \right) c_5 + 21 \left( -6 e^{2t} t + 6 e^{2t} t^2 - 2 e^{-t} t + 10 e^{2t} - 9 e^{-t} \right) c_3 + 21 \left( -3 e^{2t} t \right. \\
& + 3 e^{2t} t^2 - 4 e^{-t} t + 2 e^{2t} - 2 e^{-t} \left. \right) c_4 + 21 \left( -9 e^{2t} t + 9 e^{2t} t^2 + 8 e^{-t} t + 24 e^{2t} - 24 e^{-t} \right) c_5 \\
& + 12 \left( -13 e^{2t} + 7 e^{2t} t + \frac{7 e^{2t} t^2}{2} + 13 e^{-t} - 22 e^{-t} t \right) c_1 + 12 \left( -7 e^{2t} t + 26 e^{-t} t + 21 e^{2t} \right. \\
& - 21 e^{-t} \left. \right) c_2 + 12 \left( -7 e^{2t} t^2 + 2 e^{-t} t - 9 e^{2t} + 9 e^{-t} \right) c_3 + 10 \left( 2 e^{2t} t + 13 e^{-t} t - 8 e^{2t} \right. \\
& + 9 e^{-t} \left. \right) c_2 + 12 \left( -\frac{7 e^{2t} t^2}{2} + 4 e^{-t} t - e^{2t} + 2 e^{-t} \right) c_4 + 12 \left( -\frac{21 e^{2t} t^2}{2} - 8 e^{-t} t - 24 e^{2t} \right. \\
& + 24 e^{-t} \left. \right) c_5 + 28 \left( -e^{2t} + \frac{e^{2t} t^2}{2} + e^{-t} - 11 e^{-t} t \right) c_1 + 28 \left( -e^{2t} t + 13 e^{-t} t + 4 e^{2t} - 4 e^{-t} \right) c_2 \\
& + 28 \left( 2 e^{2t} t - e^{2t} t^2 + e^{-t} t - 5 e^{2t} + 5 e^{-t} \right) c_3 + 28 \left( e^{2t} t - \frac{e^{2t} t^2}{2} + 2 e^{-t} t - 2 e^{2t} + 2 e^{-t} \right) c_4 \\
& + 28 \left( 3 e^{2t} t - \frac{3 e^{2t} t^2}{2} - 4 e^{-t} t - 9 e^{2t} + 10 e^{-t} \right) c_5 - 19 \left( -9 e^{2t} t + 3 e^{2t} t^2 - 4 e^{-t} t - 6 e^{2t} \right. \\
& + 6 e^{-t} \left. \right) c_5 - 19 \left( -3 e^{2t} t + e^{2t} t^2 + 2 e^{-t} t - 4 e^{2t} + 4 e^{-t} \right) c_4 - 19 \left( -6 e^{2t} t + 2 e^{2t} t^2 + e^{-t} t \right. \\
& - 6 e^{2t} + 6 e^{-t} \left. \right) c_3 - 19 \left( 2 e^{2t} t + 13 e^{-t} t - 9 e^{2t} + 9 e^{-t} \right) c_2 - 19 \left( 11 e^{2t} + e^{2t} t - e^{2t} t^2 \right. \\
& - 10 e^{-t} - 11 e^{-t} t \left. \right) c_1 + 10 \left( 10 e^{2t} - e^{2t} t^2 - 10 e^{-t} - 11 e^{-t} t \right) c_1],
\end{aligned}$$

$$\begin{aligned}
& \left[ (12 e^{2t} t - 36 e^{2t} + 26 e^{-t} t - 47 e^{-t}) c_2 + 83 (-4 e^{2t} t + 2 e^{2t} t^2 + e^{-t} t - 6 e^{2t} + 6 e^{-t}) c_3 \right. \\
& + 83 (-2 e^{2t} t + e^{2t} t^2 + 2 e^{-t} t - 4 e^{2t} + 4 e^{-t}) c_4 + 83 (-6 e^{2t} t + 3 e^{2t} t^2 - 4 e^{-t} t - 6 e^{2t} \\
& + 6 e^{-t}) c_5 - 21 (13 e^{2t} - 3 e^{2t} t - 3 e^{2t} t^2 - 13 e^{-t} + 22 e^{-t} t) c_1 - 21 (6 e^{2t} t - 26 e^{-t} t - 21 e^{2t} \\
& + 21 e^{-t}) c_2 + (14 e^{2t} + 12 e^{2t} t^2 + 2 e^{-t} t + 7 e^{-t}) c_3 + (e^{2t} + 6 e^{2t} t^2 + 4 e^{-t} t - 2 e^{-t}) c_4 \\
& + (39 e^{2t} + 18 e^{2t} t^2 - 8 e^{-t} t + 32 e^{-t}) c_5 - 21 (-6 e^{2t} t + 6 e^{2t} t^2 - 2 e^{-t} t + 10 e^{2t} - 9 e^{-t}) c_3 \\
& - 21 (-3 e^{2t} t + 3 e^{2t} t^2 - 4 e^{-t} t + 2 e^{2t} - 2 e^{-t}) c_4 - 21 (-9 e^{2t} t + 9 e^{2t} t^2 + 8 e^{-t} t + 24 e^{2t} \\
& - 24 e^{-t}) c_5 + \left( -13 e^{2t} + 7 e^{2t} t + \frac{7 e^{2t} t^2}{2} + 13 e^{-t} - 22 e^{-t} t \right) c_1 + (-7 e^{2t} t + 26 e^{-t} t \\
& + 21 e^{2t} - 21 e^{-t}) c_2 + (-7 e^{2t} t^2 + 2 e^{-t} t - 9 e^{2t} + 9 e^{-t}) c_3 + (23 e^{2t} - 12 e^{2t} t - 6 e^{2t} t^2 \\
& + 35 e^{-t} - 22 e^{-t} t) c_1 + 83 (2 e^{2t} t + 13 e^{-t} t - 8 e^{2t} + 9 e^{-t}) c_2 + \left( -\frac{7 e^{2t} t^2}{2} + 4 e^{-t} t - e^{2t} \right. \\
& \left. + 2 e^{-t} \right) c_4 + \left( -\frac{21 e^{2t} t^2}{2} - 8 e^{-t} t - 24 e^{2t} + 24 e^{-t} \right) c_5 - 71 \left( -e^{2t} + \frac{e^{2t} t^2}{2} + e^{-t} \right. \\
& \left. - 11 e^{-t} t \right) c_1 - 71 (-e^{2t} t + 13 e^{-t} t + 4 e^{2t} - 4 e^{-t}) c_2 - 71 (2 e^{2t} t - e^{2t} t^2 + e^{-t} t - 5 e^{2t} \\
& + 5 e^{-t}) c_3 - 71 \left( e^{2t} t - \frac{e^{2t} t^2}{2} + 2 e^{-t} t - 2 e^{2t} + 2 e^{-t} \right) c_4 - 71 \left( 3 e^{2t} t - \frac{3 e^{2t} t^2}{2} - 4 e^{-t} t \right. \\
& \left. - 9 e^{2t} + 10 e^{-t} \right) c_5 - 58 (-9 e^{2t} t + 3 e^{2t} t^2 - 4 e^{-t} t - 6 e^{2t} + 6 e^{-t}) c_5 - 58 (-3 e^{2t} t + e^{2t} t^2 \\
& + 2 e^{-t} t - 4 e^{2t} + 4 e^{-t}) c_4 - 58 (-6 e^{2t} t + 2 e^{2t} t^2 + e^{-t} t - 6 e^{2t} + 6 e^{-t}) c_3 - 58 (2 e^{2t} t \\
& + 13 e^{-t} t - 9 e^{2t} + 9 e^{-t}) c_2 - 58 (11 e^{2t} + e^{2t} t - e^{2t} t^2 - 10 e^{-t} - 11 e^{-t} t) c_1 + 83 (10 e^{2t} \\
& - e^{2t} t^2 - 10 e^{-t} - 11 e^{-t} t) c_1 \left. \right],
\end{aligned}$$

$$\begin{aligned}
& \left[ (-14 e^{2t} t + 35 e^{2t} - 26 e^{-t} t + 47 e^{-t}) c_2 + (-14 e^{2t} t^2 - 14 e^{2t} t - 2 e^{-t} t - 7 e^{-t} - 18 e^{2t}) c_3 \right. \\
& - 82 (-4 e^{2t} t + 2 e^{2t} t^2 + e^{-t} t - 6 e^{2t} + 6 e^{-t}) c_3 - 82 (-2 e^{2t} t + e^{2t} t^2 + 2 e^{-t} t - 4 e^{2t} \\
& + 4 e^{-t}) c_4 - 82 (-6 e^{2t} t + 3 e^{2t} t^2 - 4 e^{-t} t - 6 e^{2t} + 6 e^{-t}) c_5 + 25 (13 e^{2t} - 3 e^{2t} t - 3 e^{2t} t^2 \\
& - 13 e^{-t} + 22 e^{-t} t) c_1 + 25 (6 e^{2t} t - 26 e^{-t} t - 21 e^{2t} + 21 e^{-t}) c_2 + 25 (-6 e^{2t} t + 6 e^{2t} t^2 \\
& - 2 e^{-t} t + 10 e^{2t} - 9 e^{-t}) c_3 + 25 (-3 e^{2t} t + 3 e^{2t} t^2 - 4 e^{-t} t + 2 e^{2t} - 2 e^{-t}) c_4 + 25 (-9 e^{2t} t
\end{aligned}$$

$$\begin{aligned}
& + 9 e^{2t} t^2 + 8 e^{-t} t + 24 e^{2t} - 24 e^{-t} \Big) c_5 + (-21 e^{2t} t^2 - 21 e^{2t} t + 8 e^{-t} t - 32 e^{-t} - 48 e^{2t}) c_5 \\
& + (-7 e^{2t} t^2 - 7 e^{2t} t - 4 e^{-t} t + 2 e^{-t} - 2 e^{2t}) c_4 - 82 (2 e^{2t} t + 13 e^{-t} t - 8 e^{2t} + 9 e^{-t}) c_2 \\
& + 80 \left( -e^{2t} + \frac{e^{2t} t^2}{2} + e^{-t} - 11 e^{-t} t \right) c_1 + 80 (-e^{2t} t + 13 e^{-t} t + 4 e^{2t} - 4 e^{-t}) c_2 + 80 (2 e^{2t} t \\
& - e^{2t} t^2 + e^{-t} t - 5 e^{2t} + 5 e^{-t}) c_3 + 80 \left( e^{2t} t - \frac{e^{2t} t^2}{2} + 2 e^{-t} t - 2 e^{2t} + 2 e^{-t} \right) c_4 + 80 \left( 3 e^{2t} t \right. \\
& \left. - \frac{3 e^{2t} t^2}{2} - 4 e^{-t} t - 9 e^{2t} + 10 e^{-t} \right) c_5 + 54 (-9 e^{2t} t + 3 e^{2t} t^2 - 4 e^{-t} t - 6 e^{2t} + 6 e^{-t}) c_5 \\
& + 54 (-3 e^{2t} t + e^{2t} t^2 + 2 e^{-t} t - 4 e^{2t} + 4 e^{-t}) c_4 + 54 (-6 e^{2t} t + 2 e^{2t} t^2 + e^{-t} t - 6 e^{2t} \\
& + 6 e^{-t}) c_3 + 54 (2 e^{2t} t + 13 e^{-t} t - 9 e^{2t} + 9 e^{-t}) c_2 + 54 (11 e^{2t} + e^{2t} t - e^{2t} t^2 - 10 e^{-t} \\
& - 11 e^{-t} t) c_1 - 82 (10 e^{2t} - e^{2t} t^2 - 10 e^{-t} - 11 e^{-t} t) c_1 + (-19 e^{2t} + 21 e^{2t} t + 7 e^{2t} t^2 - 35 e^{-t} \\
& + 22 e^{-t} t) c_1],
\end{aligned}$$

$$\begin{aligned}
& \left[ (-2 e^{2t} + e^{2t} t^2 + e^{2t} t - 12 e^{-t} + 11 e^{-t} t) c_1 + (-2 e^{2t} t + 7 e^{2t} - 13 e^{-t} t + 17 e^{-t}) c_2 \right. \\
& + (2 e^{2t} t - 8 e^{2t} - 2 e^{2t} t^2 - e^{-t} t - 4 e^{-t}) c_3 + (e^{2t} t - 3 e^{2t} - e^{2t} t^2 - 2 e^{-t} t) c_4 + (3 e^{2t} t \\
& - 15 e^{2t} - 3 e^{2t} t^2 + 4 e^{-t} t - 14 e^{-t}) c_5 - 24 (-4 e^{2t} t + 2 e^{2t} t^2 + e^{-t} t - 6 e^{2t} + 6 e^{-t}) c_3 \\
& - 24 (-2 e^{2t} t + e^{2t} t^2 + 2 e^{-t} t - 4 e^{2t} + 4 e^{-t}) c_4 - 24 (-6 e^{2t} t + 3 e^{2t} t^2 - 4 e^{-t} t - 6 e^{2t} \\
& + 6 e^{-t}) c_5 + 12 (13 e^{2t} - 3 e^{2t} t - 3 e^{2t} t^2 - 13 e^{-t} + 22 e^{-t} t) c_1 + 12 (6 e^{2t} t - 26 e^{-t} t - 21 e^{2t} \\
& + 21 e^{-t}) c_2 + 12 (-6 e^{2t} t + 6 e^{2t} t^2 - 2 e^{-t} t + 10 e^{2t} - 9 e^{-t}) c_3 + 12 (-3 e^{2t} t + 3 e^{2t} t^2 \\
& - 4 e^{-t} t + 2 e^{2t} - 2 e^{-t}) c_4 + 12 (-9 e^{2t} t + 9 e^{2t} t^2 + 8 e^{-t} t + 24 e^{2t} - 24 e^{-t}) c_5 + 3 \left( -13 e^{2t} \right. \\
& \left. + 7 e^{2t} t + \frac{7 e^{2t} t^2}{2} + 13 e^{-t} - 22 e^{-t} t \right) c_1 + 3 (-7 e^{2t} t + 26 e^{-t} t + 21 e^{2t} - 21 e^{-t}) c_2 + 3 ( \\
& -7 e^{2t} t^2 + 2 e^{-t} t - 9 e^{2t} + 9 e^{-t}) c_3 - 24 (2 e^{2t} t + 13 e^{-t} t - 8 e^{2t} + 9 e^{-t}) c_2 + 3 \left( -\frac{7 e^{2t} t^2}{2} \right. \\
& \left. + 4 e^{-t} t - e^{2t} + 2 e^{-t} \right) c_4 + 3 \left( -\frac{21 e^{2t} t^2}{2} - 8 e^{-t} t - 24 e^{2t} + 24 e^{-t} \right) c_5 + 29 \left( -e^{2t} + \frac{e^{2t} t^2}{2} \right. \\
& \left. + e^{-t} - 11 e^{-t} t \right) c_1 + 29 (-e^{2t} t + 13 e^{-t} t + 4 e^{2t} - 4 e^{-t}) c_2 + 29 (2 e^{2t} t - e^{2t} t^2 + e^{-t} t - 5 e^{2t} \\
& + 5 e^{-t}) c_3 + 29 \left( e^{2t} t - \frac{e^{2t} t^2}{2} + 2 e^{-t} t - 2 e^{2t} + 2 e^{-t} \right) c_4 + 29 \left( 3 e^{2t} t - \frac{3 e^{2t} t^2}{2} - 4 e^{-t} t \right.
\end{aligned}$$

$$\begin{aligned}
 & -9e^{2t} + 10e^{-t}) c_5 + 14(-9e^{2t}t + 3e^{2t}t^2 - 4e^{-t}t - 6e^{2t} + 6e^{-t}) c_5 + 14(-3e^{2t}t + e^{2t}t^2 \\
 & + 2e^{-t}t - 4e^{2t} + 4e^{-t}) c_4 + 14(-6e^{2t}t + 2e^{2t}t^2 + e^{-t}t - 6e^{2t} + 6e^{-t}) c_3 + 14(2e^{2t}t \\
 & + 13e^{-t}t - 9e^{2t} + 9e^{-t}) c_2 + 14(11e^{2t} + e^{2t}t - e^{2t}t^2 - 10e^{-t} - 11e^{-t}t) c_1 - 24(10e^{2t} \\
 & - e^{2t}t^2 - 10e^{-t} - 11e^{-t}t) c_1]
 \end{aligned}$$

> **simplify(%);**

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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